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CURRENT TRENDS TO THE USE OF SWITCHING SUPERVISORY LOGIC IN AUTOMATIC CONTROL OF UNCERTAIN OR CONSTRAINED DYNAMIC SYSTEMS

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Abstract

A current trend in the design of control systems consists of replacing classic techniques of *gain-scheduling* (where the switching mechanism is open-loop) with *switching supervisory logic* schemes based on actual experimental data (and, hence, of closed-loop nature). Such a replacement can yield dramatic improvements in performance, reliability and safety, particularly in the presence of model uncertainties or constraints.

Two different exemplifications of switching supervisory control will be presented: One concerning regulation under control saturation constraints; and, the second, unfalsified adaptive control of unknown dynamic systems.

Combinando nei sistemi di controllo logiche di supervisione con elementi di AI si sono conseguiti obiettivi rimasti a lungo privi di soluzioni *concettualmente* soddisfacenti.

Es. ***CONTROLLO ADATTATIVO***: Dato un sistema dinamico *incerto* (impianto) *P* individuare da soli dati di I/O di *P* un'unità di controllo *C* che posta in retroazione con *P* consenta di assicurare al sistema di controllo risultante prestazioni soddisfacenti.

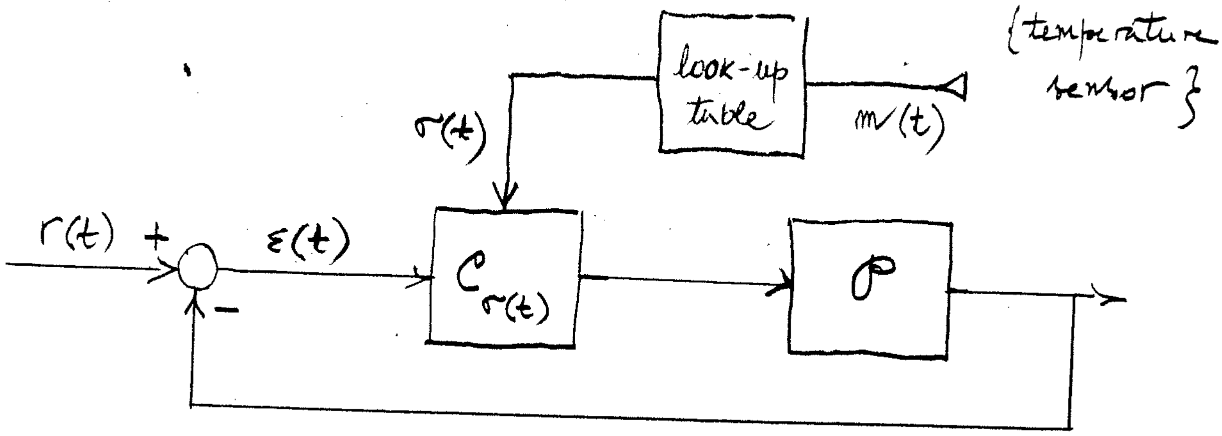
Un obiettivo questo era venuto ad emergere per impianti *P incogniti* alla fine degli anni 60 culminando anche in qualche disastroso esperimento di volo.

Dall'inizio degli anni 70 un obiettivo più realistico (per impianti con incertezze circoscritte) era stato riavviato con intensità e significativi successi consistenti principalmente in rigorosi risultati analitici. Anche esso, tuttavia, all'inizio degli anni 80 era culminato in incertezze e disillusioni a fronte della scoperta di contro-esempi che ne denunciavano l'assenza di robustezza delle soluzioni raggiunte.

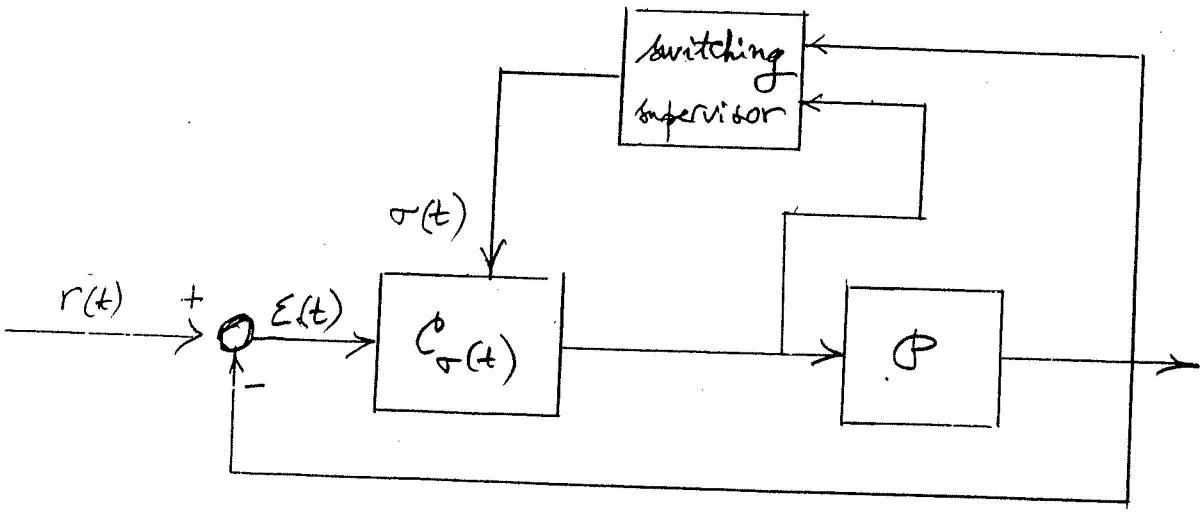
La successiva involuzione della ricerca accademica è andata mediamente manifestandosi con forme sempre più astratte di analisi subordinate ad ipotesi spesso irrealistiche o almeno non verificabili.

<u>Avvertenza</u> - Non si intende rivendicare una sicura praticità applicativa di quanto sarà esposto, ma piuttosto il valore di risposta "*concettualmente definitiva*" in grado di stimolare necessari ulteriori approfondimenti per concrete applicazioni.

Gain-scheduling vs. switching supervisory control



the switching mechanism is open-loop



the switching mechanism is closed-loop

Adaptive S.S.C.

"*Switched system*":

$$x(t+1) = f_{\sigma(t)}(x(t), t)$$

$t \in \mathbb{Z}_+ := \{0, 1, \cdots\}$

$\sigma(t) \in \overleftarrow{N} := \{1, 2, \cdots, N\}$

Ex. 1 (rif. e disturbi costanti a tratti)

$$\begin{cases} \chi(t+1) = \varphi \chi(t) + g u(t) + \xi \\ y(t) = h \chi(t) + \zeta \\ \varepsilon(t) = y(t) - r \end{cases}$$

$$\delta u(t) := u(t) - u(t-1)$$

$$P : \begin{cases} \delta\chi(t+1) = \varphi \delta\chi(t) + g \delta u(t) \\ \varepsilon(t) = \varepsilon(t-1) + h \delta\chi(t) \end{cases} \sim \quad x(t+1) = \Phi x(t) + G \delta u(t)$$

$$x(t) := [\delta\chi'(t) \;\; \varepsilon'(t-1)]'$$

under a state-feedback control (one among N candidate feedback-gains)

[prime denotes transpose].

$$\mathcal{C}_{\sigma(t)}: \quad \delta u(t) = -F^{\chi}_{\sigma(t)}\, \delta\chi(t) - F^{\varepsilon}_{\sigma(t)}\, \varepsilon(t-1)$$

$$= -F_{\sigma(t)}\, x(t)$$

$$F_{\sigma(t)} := \begin{bmatrix} F^{\chi}_{\sigma(t)} & F^{\varepsilon}_{\sigma(t)} \end{bmatrix}$$

$(P/C_{\sigma(t)})$ denotes the "switched system"

$$x(t+1) = \Phi_{\sigma(t)}\, x(t)$$

$$\Phi_{\sigma(t)} := \Phi - \mathcal{G} F_{\sigma(t)}$$

For each candidate controller $\mathcal{C}_i$, $i \in \overleftarrow{N}$, a "test functional" is defined

$$V_i(t) = g_i(x^t, t) \geq 0$$

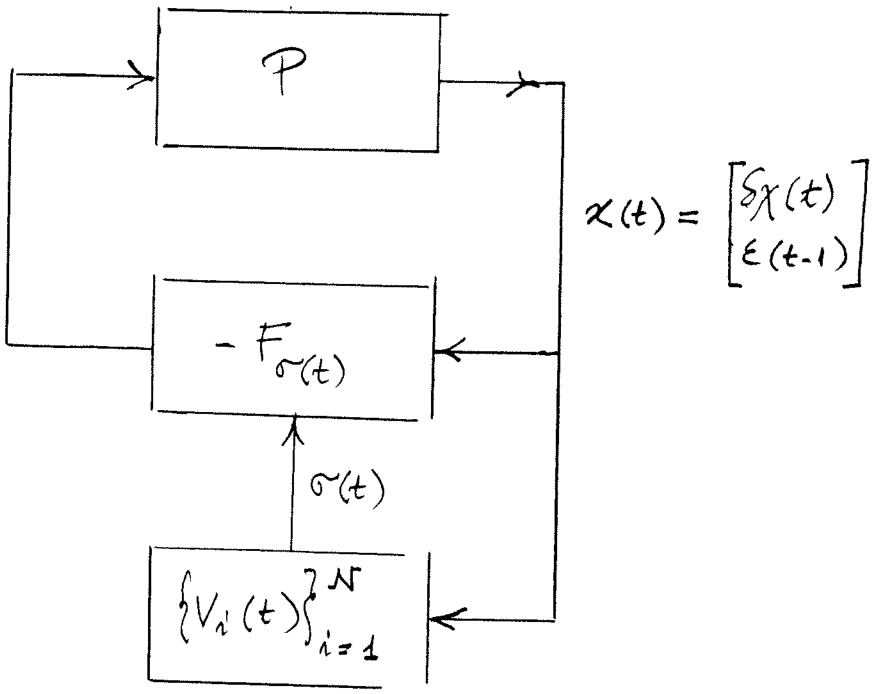
$$x^t := \{x(\tau)\}_{\tau=0}^{t} \in X^{t+1}$$

for every possible initial state $x(0)$ and switching sequence

$$\sigma^t = \{\sigma(\tau)\}_{\tau=0}^{t}$$

The meaning of $V_i(t)$ is that it quantifies the loss achieved by the use of $\mathcal{C}_i$ associated to the experimental data x^t. The smaller $V_i(t)$, the greater the expected performance achievable by $\mathcal{C}_i$.

Summing up:



<u>Hysteresis switching supervisory logic</u>:

$$\sigma(t) = \begin{cases} \sigma(t-1), & V_{\sigma(t-1)}(t) < V_{\hat{\jmath}(t)}(t) + h \\ \hat{\jmath}(t) & \text{otherwise} \end{cases}$$

$\hat{\jmath}(t) :=$ the least integer s.t.

$$= \arg\min_i \{ V_i(t),\ i \in \overleftarrow{N} \}$$

$\sigma(0) = \sigma_0 \in \overleftarrow{N}$

$h > 0$ (& small) the hysteresis constant

HSSL Lemma (Mayne, Morse, Godwin, IE^3 TAC, 1992)

$\mathcal{S}$: the set of all possible switching sequences

$s : \mathbb{Z}_+ \to \overleftarrow{N}$

$x_s(\cdot)$: the solution of $\quad x(t+1) = f_{s(t)}(x(t), t)$

$x(0) = x_0$

A1) $\forall s \in \mathcal{S}$ & $\forall i \in \overleftarrow{N}$,

$$\exists \lim_{t\to\infty} V_i(t) = \lim_{t\to\infty} g_i(x_s^t, t) \in \{\mathbb{R}_+, \infty\}$$

A2) $\exists n \in \overleftarrow{N}$ s.t.

$V_n(t) = g_n(x_s^t, t)$ is bounded.

Then,

A1) & A2) $\Rightarrow$ $\exists t^* \in \mathbb{Z}_+$ s.t., for the switched system subject to the HSSL,

$\sigma(t) = \sigma(t^*)$, $\forall t \geq t^*$

&

$V_{\sigma(t^*)}(\cdot)$ is bdd

The HSSL Lemma found a significant use only 15 years after its publication! (Safonov, 2006)

Ex. (cont.): $\quad x(t+1) = \Phi x(t) + G\,\delta u(t)$

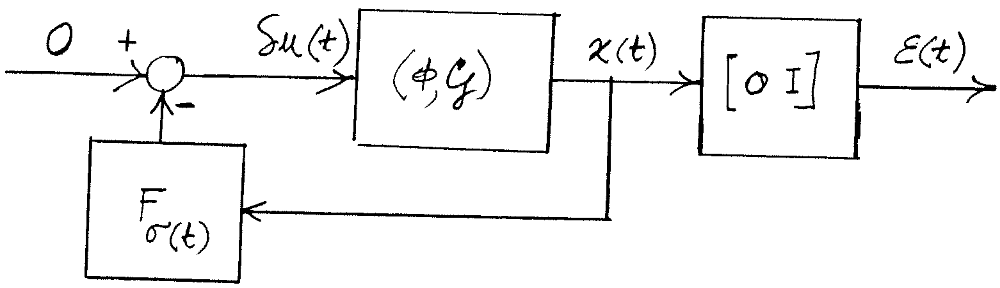
$$\delta u(t) = -F_{\sigma(t)}\, x(t)$$

Define the ith virtual reference $v_i(t)$ as follows:

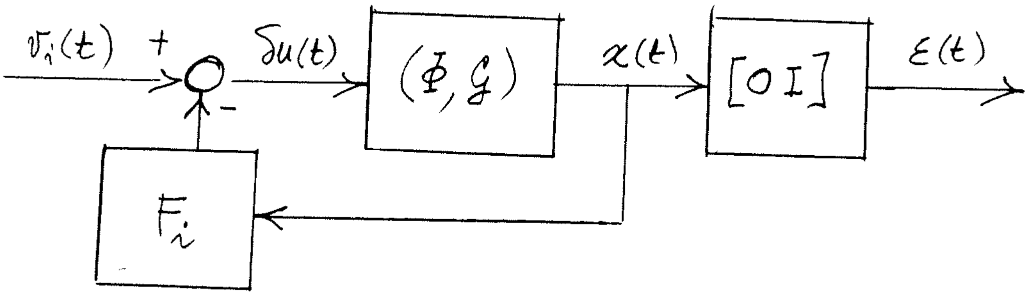
$$v_i(t) := \delta u(t) + F_i\, x(t)$$

$$= \left(F_i - F_{\sigma(t)}\right) x(t)$$

"Actual experiment"



"i-th virtual experiment"



$v_i(\cdot)$ is the hypothetical disturbance which, if injected into (P/F_i) would generate the same I/S variables as the ones recorded from the actual loop $(P/F_{\sigma(\cdot)})$

Remark - The i-th virtual experiment can be used so as to assess the performance of (P/F_i), $P=?$, without switching on in feedback with the real plant the state feedback-gain F_i under test.

Ex. (Cont.) : A way for defining $V_i(t)$

Set $(\rho, c^2 > 0)$

$$J_i(t) = \gamma_i(x^t, t) := \frac{\| \varepsilon^t \|^2 + \rho \, \| \delta u^t \|^2}{c^2 + \| v_i^t \|^2}$$

"loss" up to time t

$$\| \varepsilon^t \|^2 := \sum_{k=0}^{t} | \varepsilon(k) |^2 \, , \qquad | \varepsilon(k) | \text{ Euclidean norm}$$

In words, the larger the loss that pertains to (P/F_i), $P=?$, for given recorded data δu^t, x^t, the smaller $\| v_i^t \|^2$. I.e., the higher the ℓ_{2e}-ℓ_{2e} gain from the disturbance v_i^t to the recorded data.

Further, the i-th test functional ($t = 1, 2, \cdots$)

$$V_i(t) := \max\{J_i(\tau),\ 0 \le \tau \le t-1\}$$
$$=: \max J_i^{t-1}$$

$V_i(t)$ is the largest loss J_i over $[0, t-1]$.

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The V_i's satisfy A1) as they are monotone.

They also satisfy A2) under the "feasibility assumption":

$\exists\, m \in \overleftarrow{N}$ s.t. (P/F_m) is stable.

$\Longrightarrow$ the HSSL lemma holds

$\Longrightarrow \exists\, t_f \in \mathbb{Z}_+$ s.t. $F_{\sigma(t)} = F_f, \quad \forall t \ge t_f$

& $V_f(\cdot)$ bdd $\Longrightarrow$ $\varepsilon(k) \to 0$
$\delta u(k) \to 0$

$$\left\{ F_{\sigma(t)} = F_f \Longrightarrow \sigma_f(t)\Big|_{t \ge t_f} = 0 \right\}$$

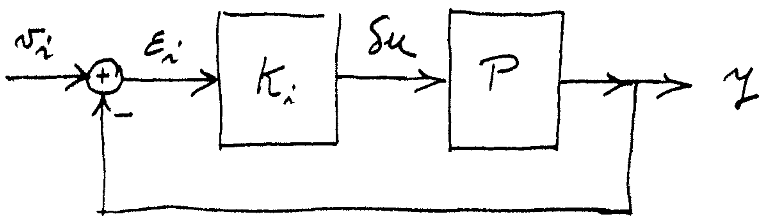
Dynamic compensation

need not be linear

$$(P/K_{\sigma(\cdot)}) \sim \begin{cases} y(t) = P(\delta u)(t) \\ \delta u(t) = K_{\sigma(t)}(r-y)(t) \end{cases}$$

The K_i's are causal & stably causally invertible

virtual ref. $\rightarrow$ $v_i(t) = y(t) + K_i^{-1}(\delta u)(t)$



$$J_i(t) = \gamma_i(\delta u^t, y^t)$$

$$:= \frac{\|\varepsilon_i^t\|^2 + \rho\,\|\delta u^t\|^2}{c^2 + \|v_i^t\|^2}$$

$$V_i(t) := \max J_i^{t-1}$$

"feasibility assumption" $\Longrightarrow$ HSSL lemma holds &

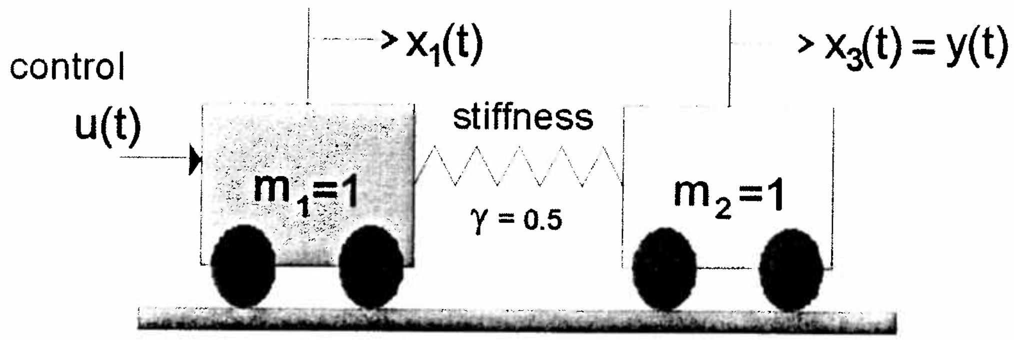
$$\|y^t\| \le \alpha + \beta \|r^t\|$$

$$\|\delta u^t\| \le \gamma + \delta \|r^t\|$$

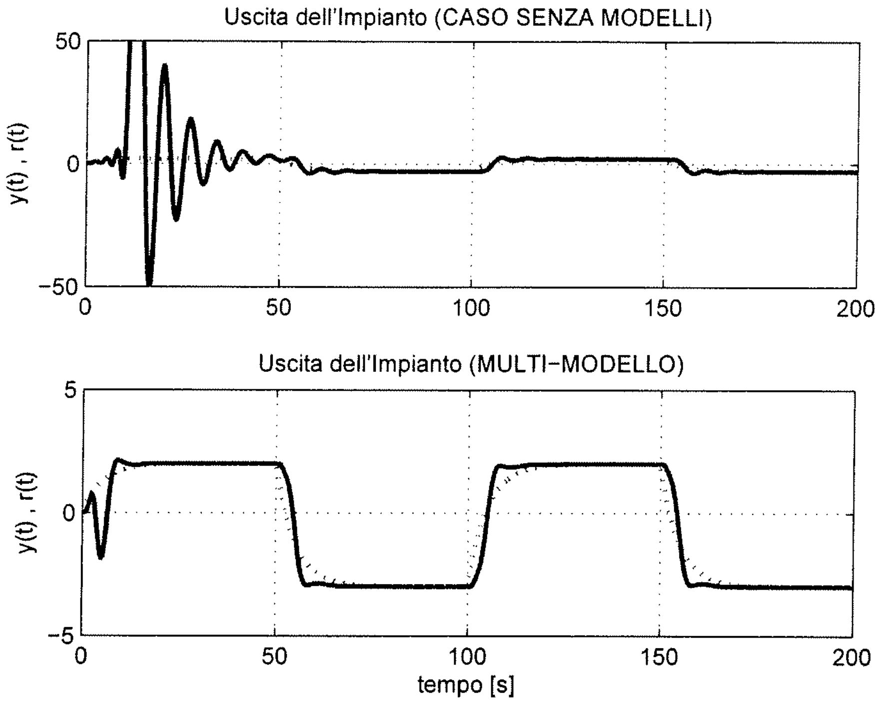
On-going extentions

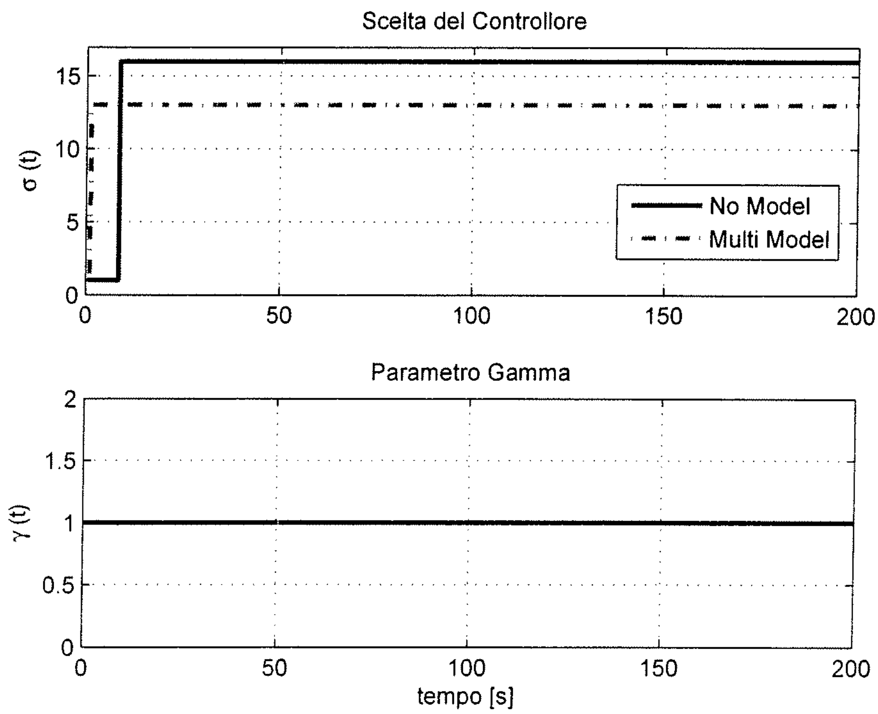
- Multi-model VRASSC.
- Analysis under generic disturbances.
- Time-varying case.

Switching horizon predictive control: Example 1

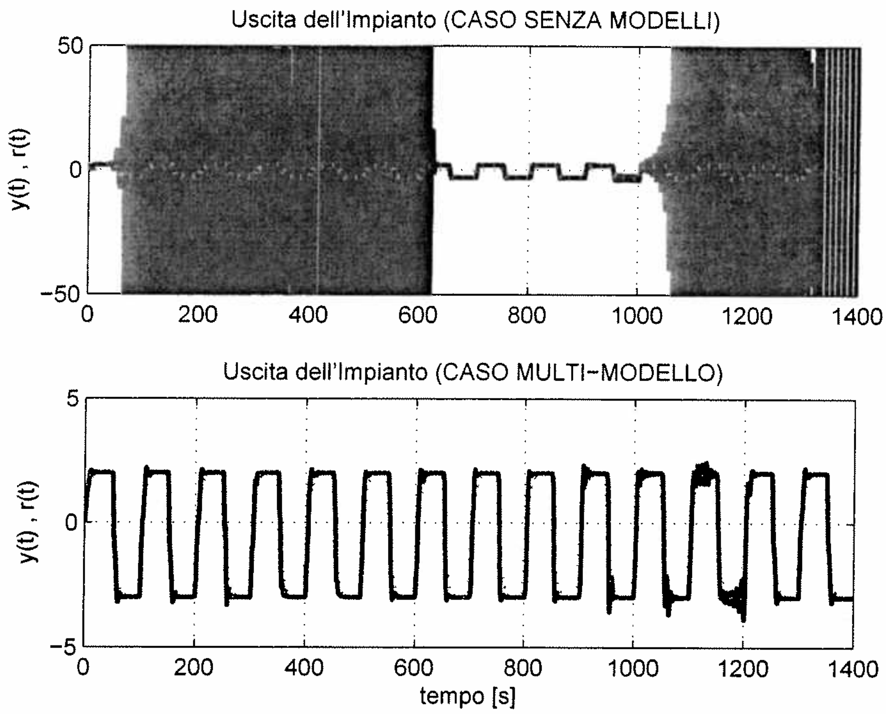


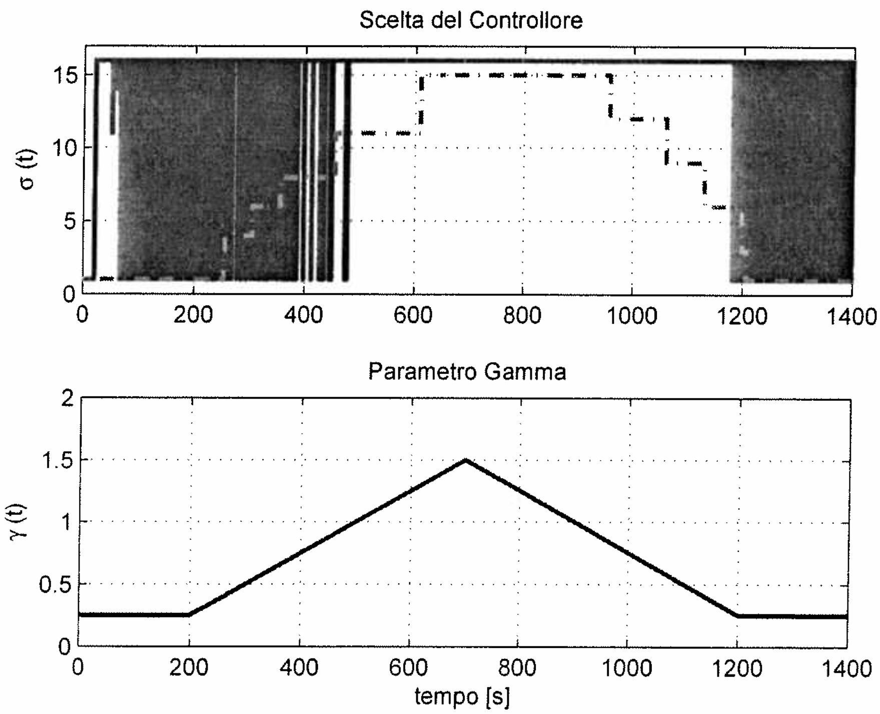
Caso Tempo-Invariante (γ cost., $N = 16$)





Caso Tempo-Variante ($N = 16,\ \lambda = 0.995$)





Caso Tempo-Variante ($N = 16,\ \lambda = 0.95$)