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*"La mente è come un paracadute.*
*Funziona solo se si apre"*
*A. Einstein*

## Quadratization of Pseudo-Boolean Functions

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Abstract

The problem of minimizing a pseudo-Boolean function (over the set of binary vectors) appears to be the common form of numerous optimization problems, including the well-known MAX-SAT and MAX-CUT problems, and have applications in areas ranging from physics through chip design to computer vision.

Some of these applications lead to the minimization of a quadratic pseudo- Boolean function, and hence such quadratic binary optimization problems received ample attention in the past decades. One of the most frequently used technique is based on roof-duality (Hamer, Hansen, Simeone, 1984), and aims at finding in polynomial time a simpler form of the given quadratic minimization problem, by fixing some of the variables at their provably optimum value (persistency) and decomposing the residual problem into variable disjoint smaller subproblems (Boros and Hammer, 1989). The method in fact was found very effective in computer vision problems, where frequently it can fix up to 80-90% of the variables at their provably optimum value (Boros, Hammer, Sun and Tavares, 2008). This algorithm was used by computer vision experts and a very efficient implementation, called QPBO, is freely downloadable (Rother, Kolmogorov, Lempitsky and Szummer, 2007).

In many applications of pseudo-Boolean optimization, in particular in vision problems, the objective function is a higher degree multilinear polynomial. For such problems there are much fewer effective techniques available. In particular, there is no analogue to the persistencies (fixing variables at their provably optimum value) provided by roof-duality for the quadratic case. On the other hand, more and more applications would demand fast computational methods for the minimization of such higher degree pseudo-Boolean functions. This increased interest, in particular in the computer vision community, lead to a systematic study of methods to reduce a higher degree minimization problem to a quadratic one. There are many techniques to achive this goal, most of them based on local transformations.

We follow a global approach, considering all possible quadratizations of a given higher degree pseudo-Boolean function, and provide sharp lower and upper bounds on the number of extra variables needed to quadratize those. In fact we consider several diferent types of quadratizatons, and provide sharp bounds for each. These results give rise to new quadratization algorithms, as well. We report on some computational results which we got via a collaboration with Alex Fix and Ramin Zabih (Cornell University, NY, USA).

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