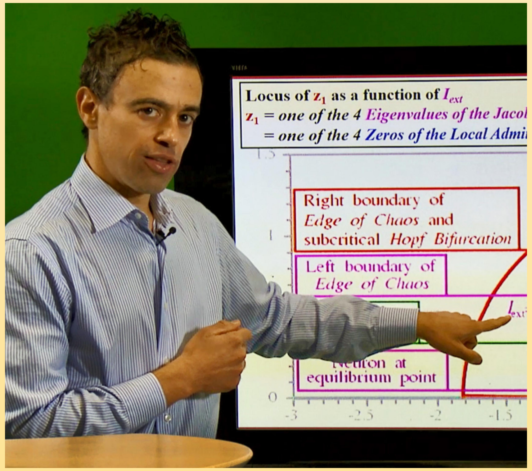
Local Activity and Edge of Chaos in NaMLab Memristive Devices and Circuits

Alon Ascoli



Chair of Fundamentals of Electrical Engineering, Institute of Circuits and Systems, Faculty of Electrical and Computer Engineering, Technische Universität, Dresden, Germany

Invited Seminar

IASI CNR

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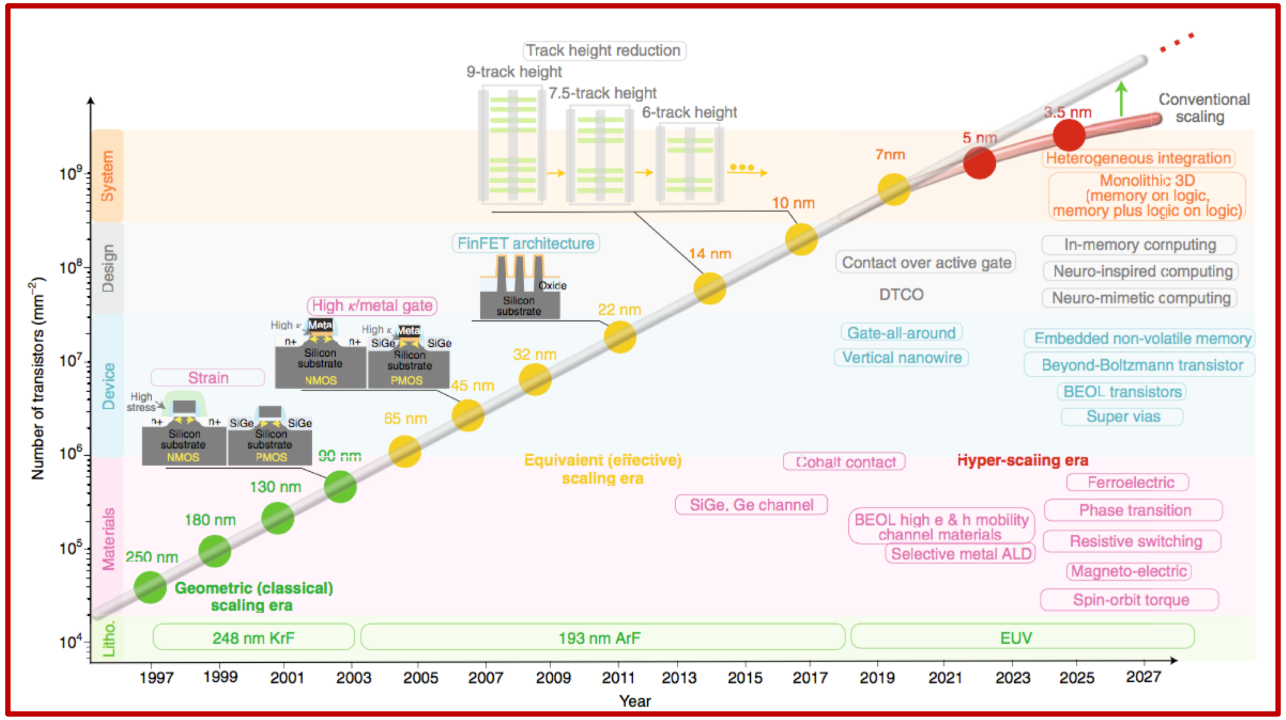
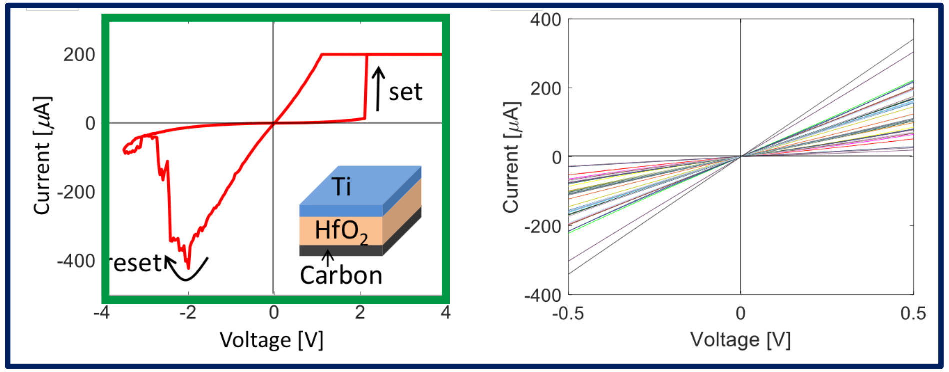
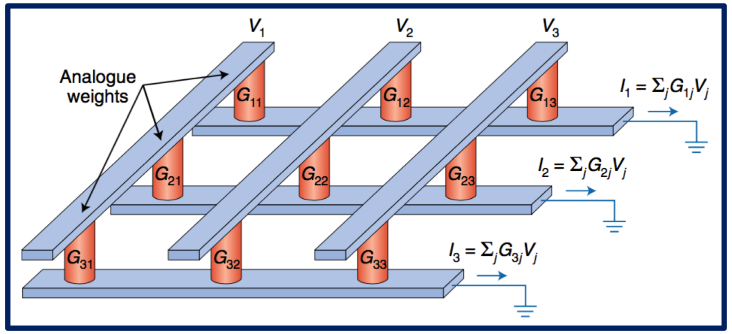
The Key Technology to enable IC performance growth beyond the Moore's Era



S. Salahuddin, K. Ni, and S. Datta, "The era of hyper-scaling in electronics," Nature Electronics, vol. 1, no. 8, pp. 442–450, 2018



Z. Sun, G. Pedretti, E. Ambrosi, A. Bricalli, W. Wang, and D. Ielmini, "Solving matrix equations in one step with cross-point resistive arrays," Proceedings of the National Academy of Sciences, vol. 116, no. 10, pp. 4123–4128, 2019.



D. Ielmini and H. S. P. Wong, "In-memory computing with resistive switching devices," Nature Electronics, vol. 1, no. 6, pp. 333–343, 2018.

- ***Shrinking transistor sizes shall no longer be feasible in the years to come***
  - ***The disruptive memristor technologies allow to fabricate nanodevices with state-dependent resistances, capable to sense, store, and compute data in the very same physical medium.***
  - ***Memristors enable the hardware implementation of novel bio-inspired data processing paradigms, e.g. in-memory computing, allowing to boost the performance of purely-CMOS hardware***
- ***Nonlinear circuit and system theory assumes a primary role in the development of robust IC designs with inherently-nonlinear memristor devices***

Volatile Memristor Memories with *Small-Signal Amplification* Capability

- *The most general definition of a n^{th} order current-controlled memristor $\mathcal{M}$ is a Differential Algebraic Equation (DAE) set of the form*

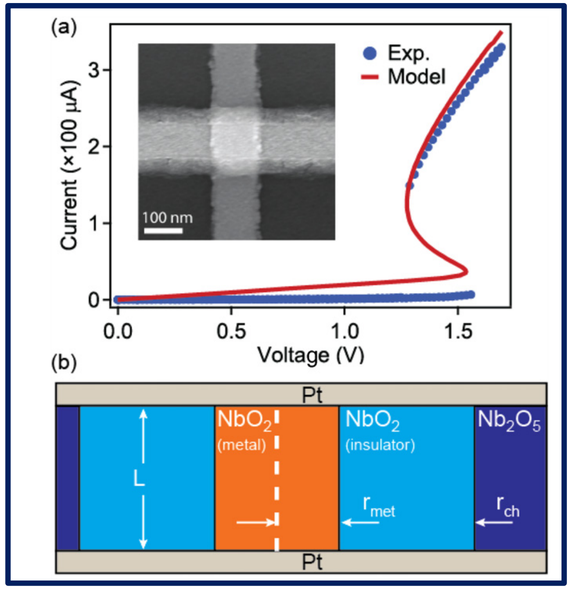
$$\begin{cases} \frac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(\boldsymbol{x}, i) \\ v = v(\boldsymbol{x}, i) = R(\boldsymbol{x}, i) \cdot i, \text{ with } \lim_{i \to 0A} R(\boldsymbol{x}, i) \neq \infty \end{cases}$$

STATE EQUATION OF AN EXTENDED MEMRISTOR

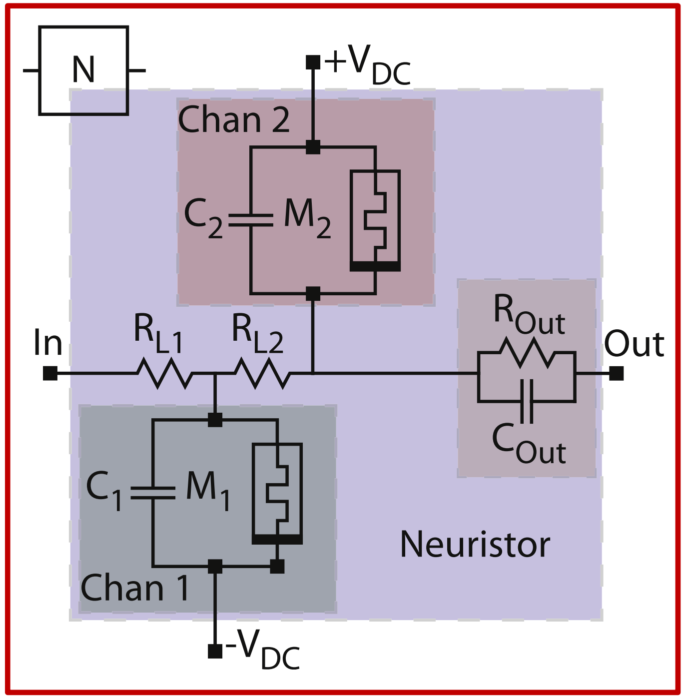
OHM'S LAW OF AN EXTENDED MEMRISTOR

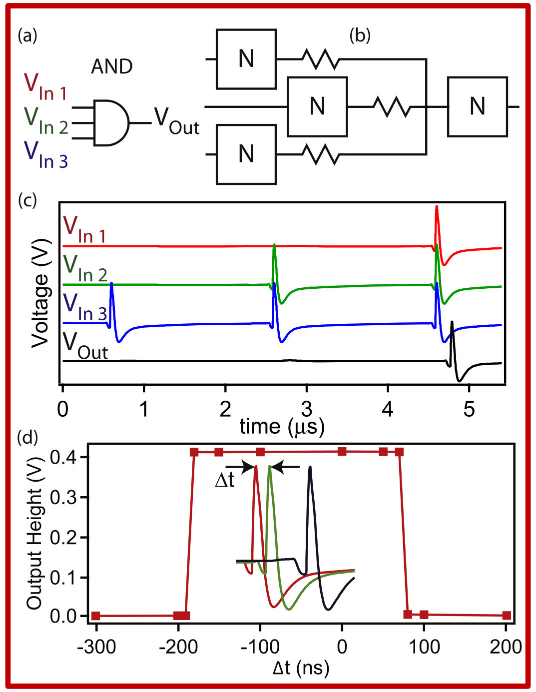
where $\boldsymbol{x} = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$ is a n-dimensional state, and $R(\boldsymbol{x}, i)$ is a state- and input-dependent resistance

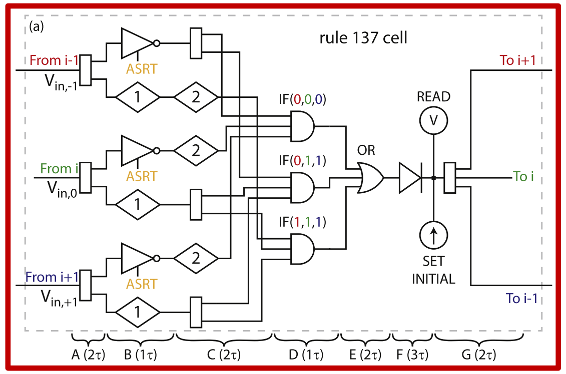
- *If, turning the power off, $R(x(t), i = 0A)$ is found to converge toward one state from an analogue continuum or toward one of a few (at least two) isolated states, depending upon the initial condition $\boldsymbol{x}(0)$, the memristor is said to be non-volatile*
- *Among the class of volatile memristors, those featuring a Negative Differential Resistance (NDR) region in their DC current-voltage characteristic are of great interest* ⇒ *main application: development of brain-like computing machines*

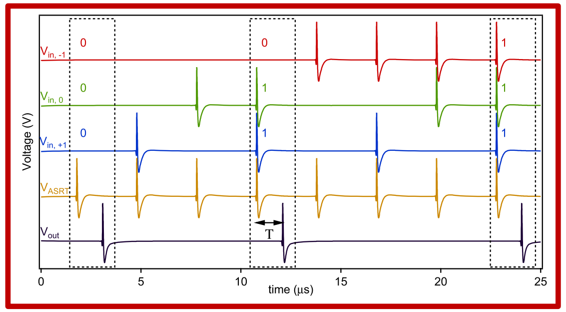


M. D. Pickett and R. S. Williams, "Sub-100 fJ and sub-nanosecond thermally driven threshold switching in niobium oxide crosspoint nanodevices," Nanotechnology, 23, 215202, 2012







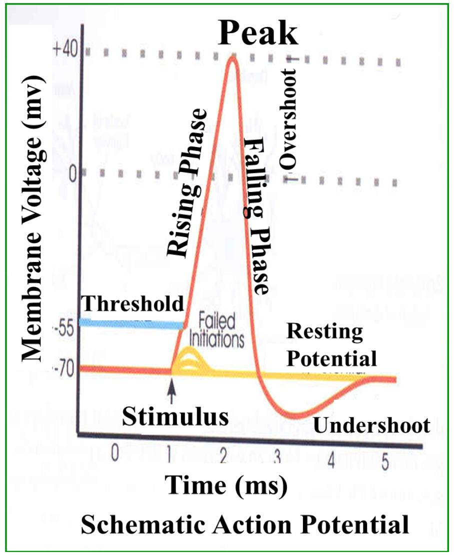


M. D. Pickett and R. S. Williams, "Phase transitions enable computational universality in neuristor-based cellular automata," Nanotechnology, 24, 384002, 2013

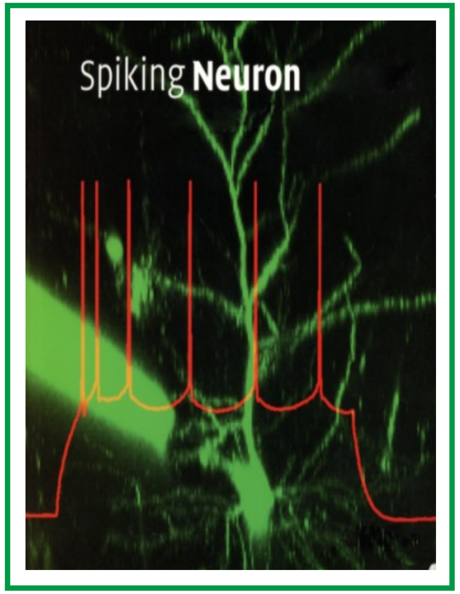
The Origin for Complexity

- ***Volatile Memristors with NDR effects induce complex bio-mimetic phenomena, e.g. the emergence of an All-or-None Spike reminiscent of neuronal dynamics, in electrical circuits***

(a)



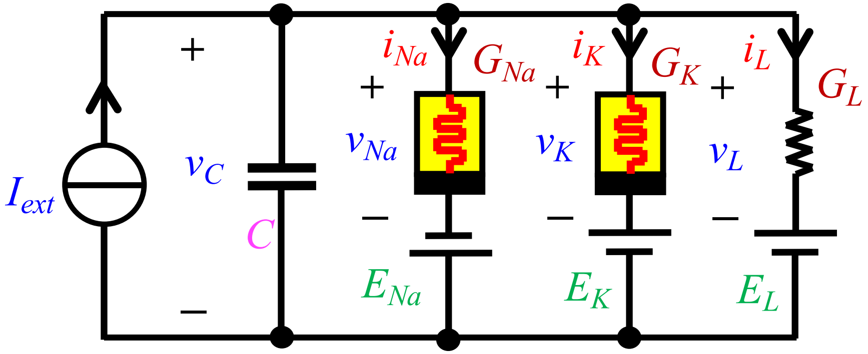
(b)



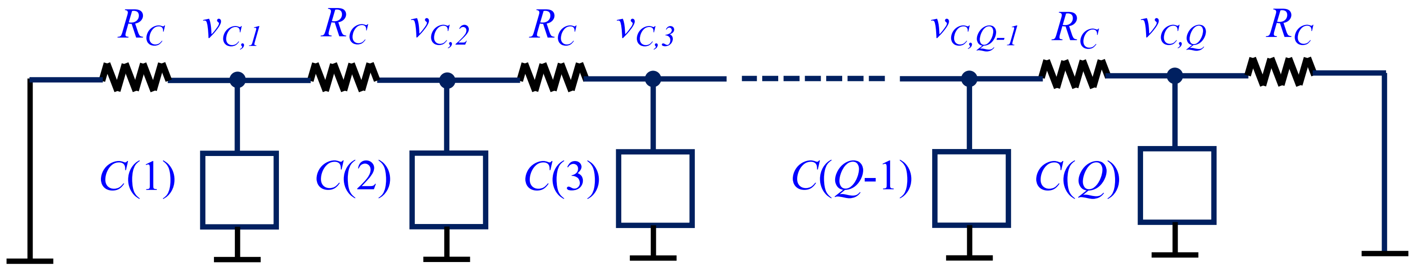
Sketch of an Action Potential (a) and of a Spiking Neuron (b)

- ***But how does complexity originate? What are the necessary conditions for a physical system to exhibit emergent phenomena?***

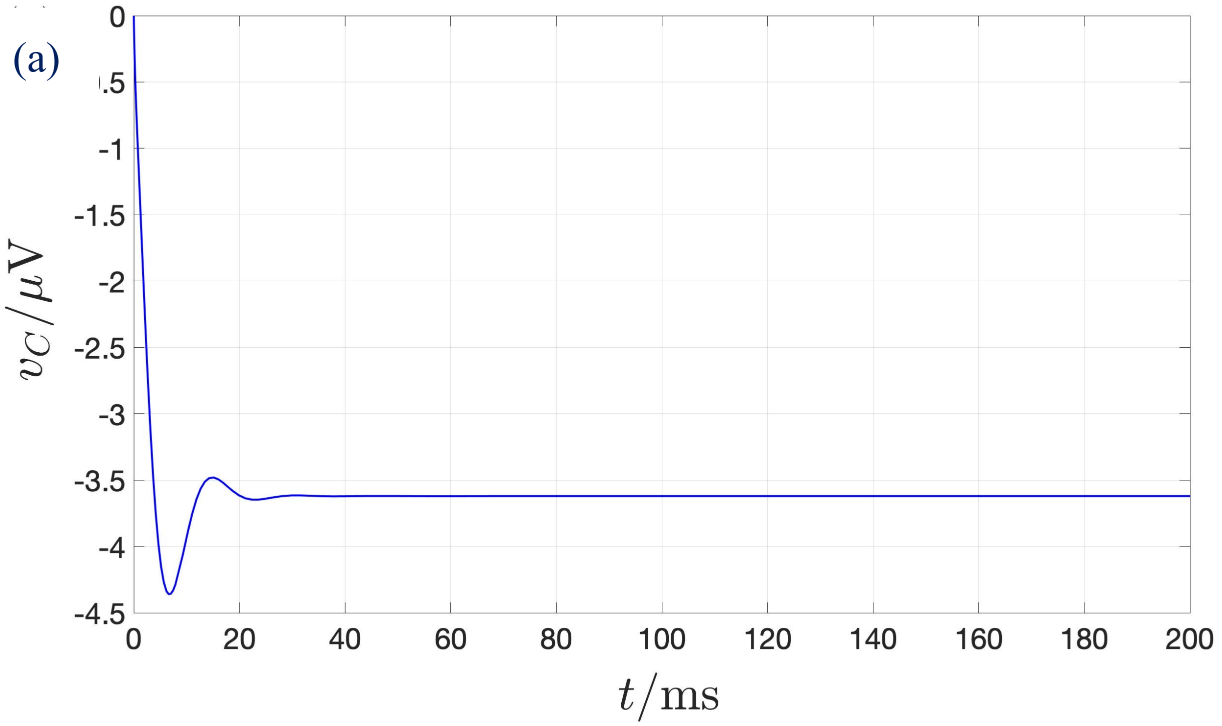
Illustrative scenario – Action Potential Propagation in a Chain of Neurons



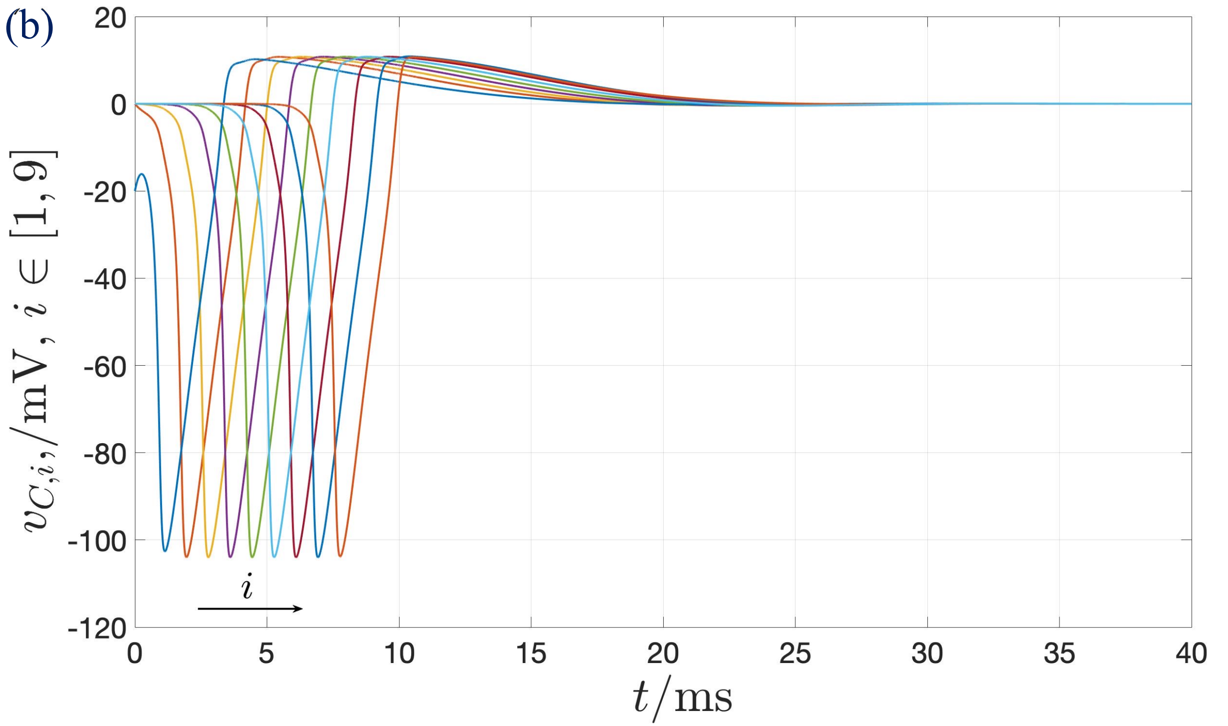
Hodgkin-Huxley (HH) [hod52] neuron axon membrane circuit model



One-dimensional array of identical diffusively-coupled HH neurons



(a) Time course of the capacitor voltage of the isolated cell for $I_{ext} = 0\text{A}$

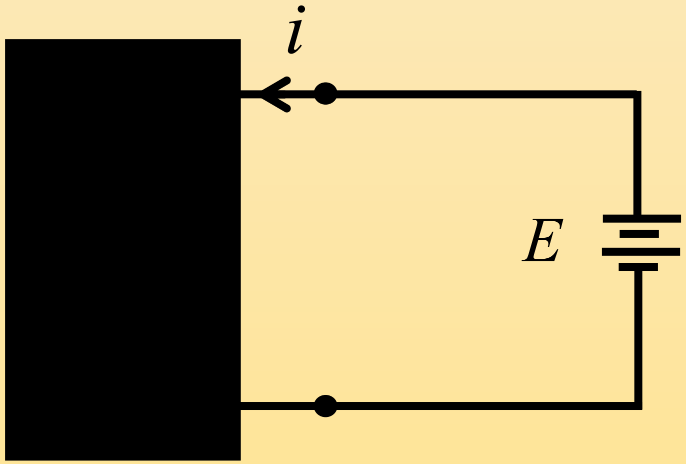


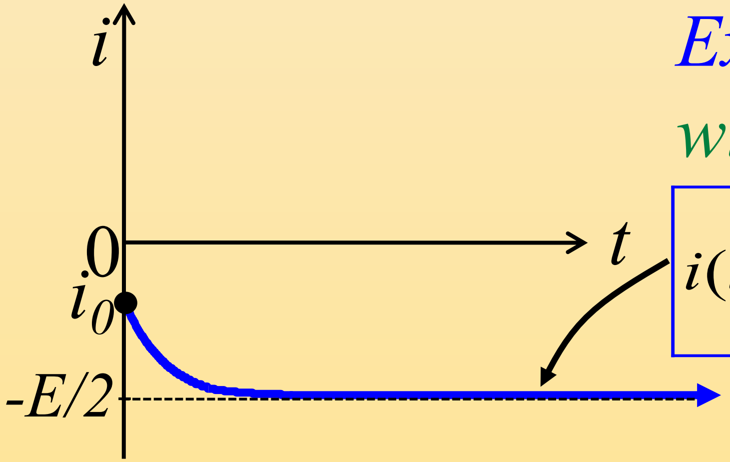
(b) Time course of the capacitor voltage of each of the $Q = 9$ identical cells of the array for $I_{ext} = 0\text{A}$. All cells except C(1) share the same initial condition as in (a)

A.L. Hodgkin, and A.F. Huxley, "A quantitative description of membrane current and its application to conduction and excitation in nerve," J. Physiol., vol. 117, pp. 500–544, 1952

Chua's Riddle

Chua's Riddle

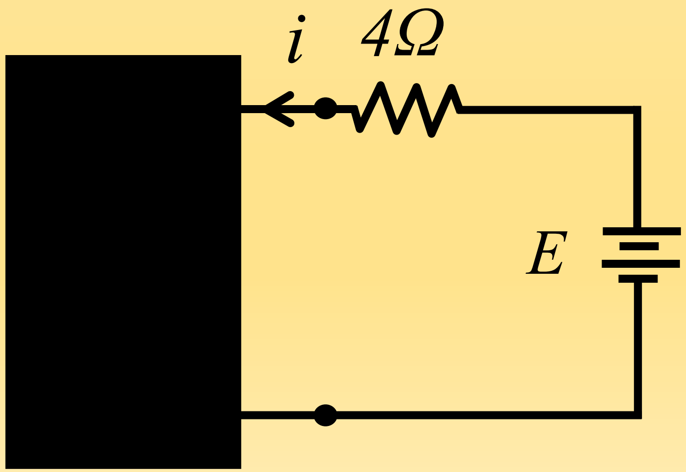


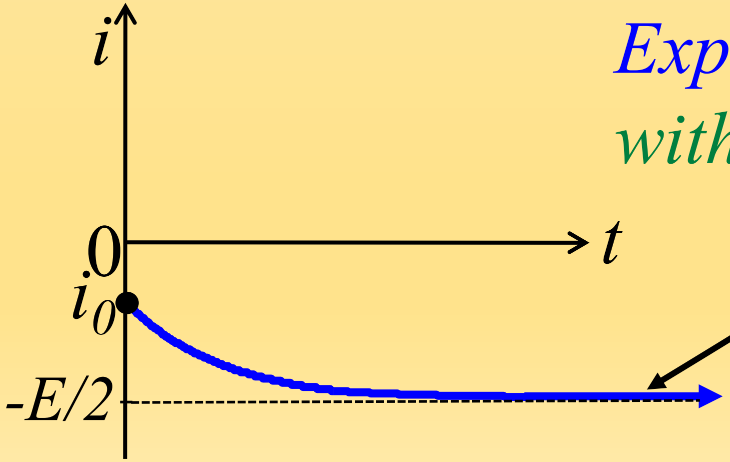


Exponential waveform

with time constant $\tau = 1$

$$i(t) = -\frac{E}{2} + \left[i_0 + \frac{E}{2} \right] e^{-t}$$

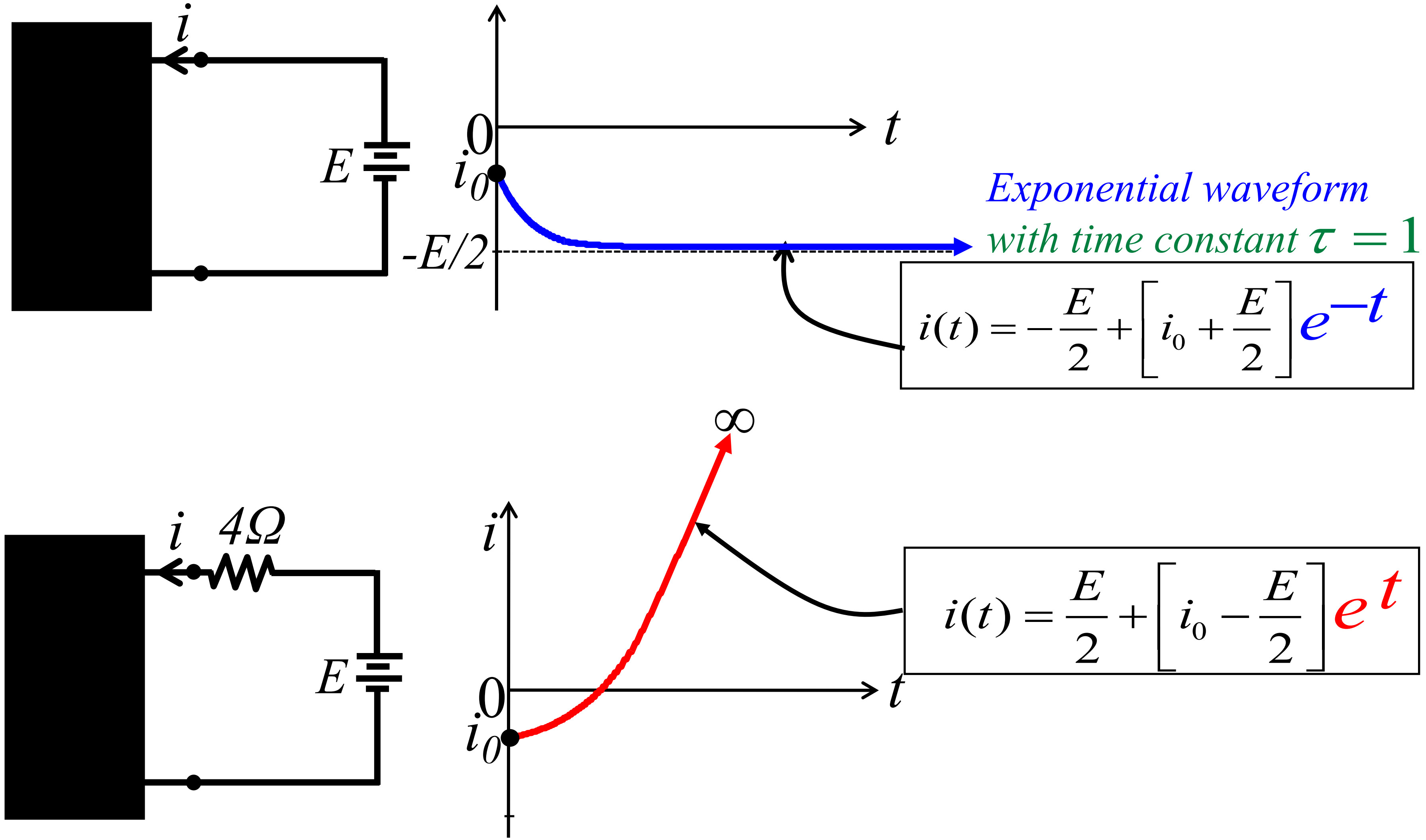




Exponential waveform

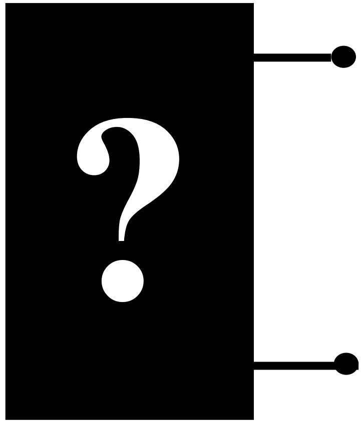
with time constant $\tau > 1$

Chua's Riddle

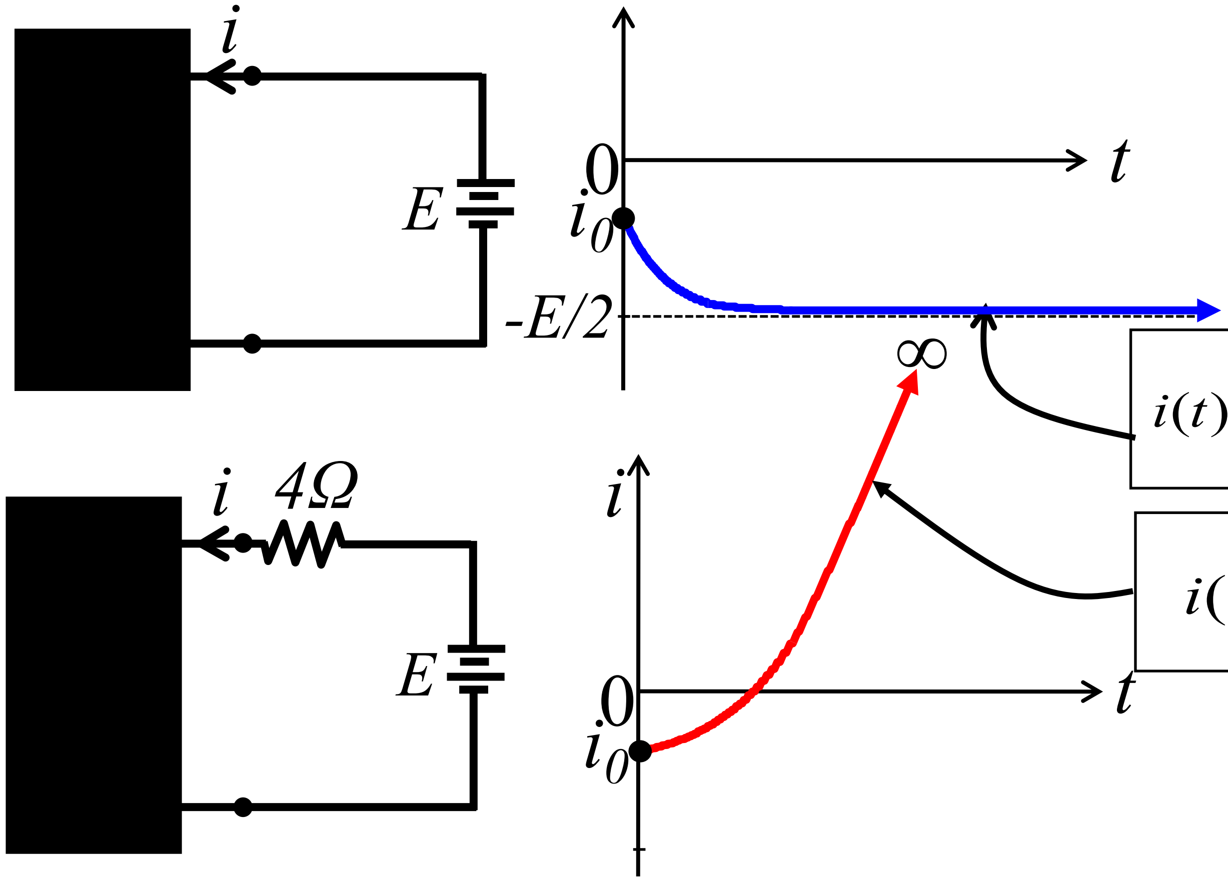


- ***This is yet another example of Complexity. How may this happen?***

What is inside the Black Box



Chua's Riddle



Exponential waveform with time constant $\tau = 1$

$$i(t) = -\frac{E}{2} + \left[i_0 + \frac{E}{2} \right] e^{-t}$$

$$i(t) = \frac{E}{2} + \left[i_0 - \frac{E}{2} \right] e^{t}$$

Hint : the Black Box contains just two basic linear two-terminal circuit elements

The *Answer*
to
Chua's Riddle
is the Essence of the
Edge of Chaos

Schrödinger, Prigogine, Eigen,

Gell-Mann, Turing, and Smale

have all been searching for a *missing*

new Physics Principle

to explain Complexity

in physical systems

The Local Activity

and its Pearl,

the Edge of Chaos,

is in fact the

Missing New Principle

Local Activity Principle

Complex and Emergent phenomena are impossible without Local Activity

Necessary Conditions for Complexity

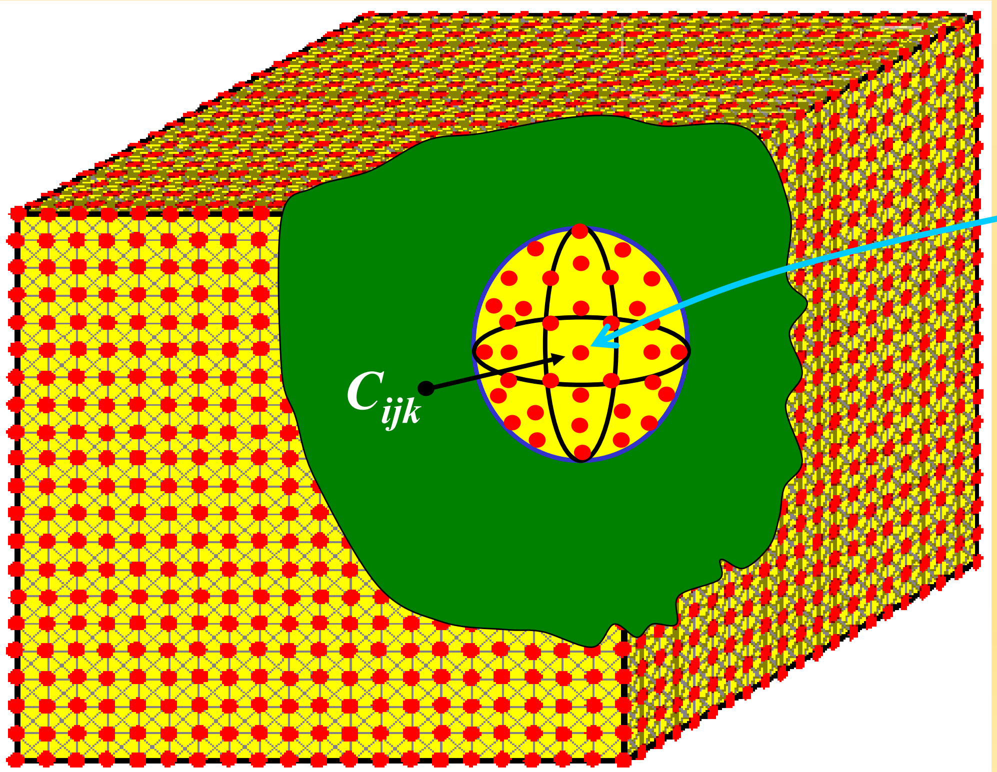
Erwin Schrödinger :

- External Supply of energy

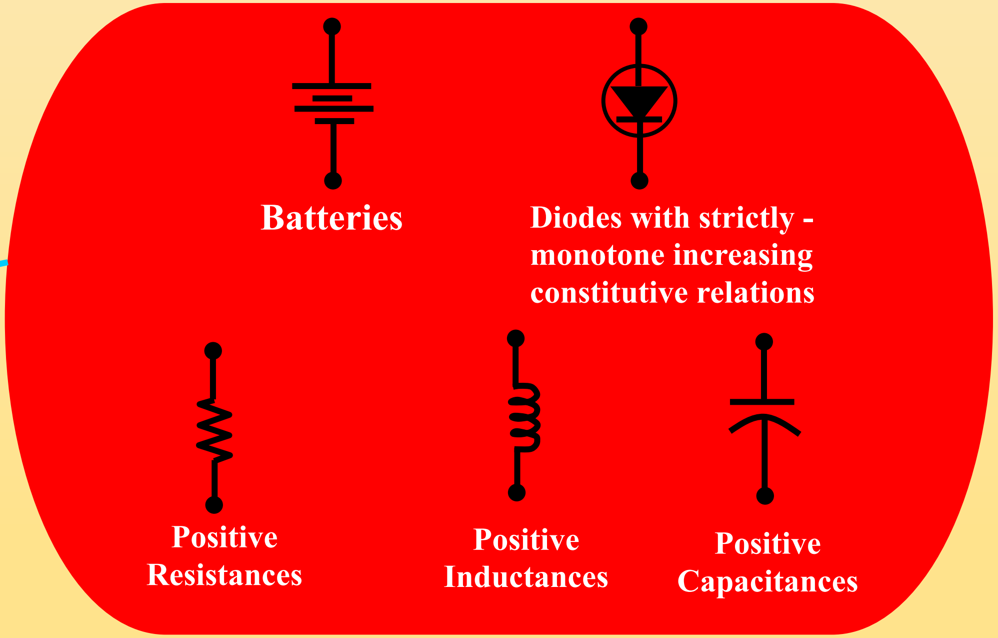
Ilya Prigogine :

- Presence of nonlinearity

- *Each cell of the network is made of arbitrary interconnections of electrical elements endowed with the two aforementioned conditions for Complexity*
- *The coupling circuit is made up of positive linear resistors*
- *This would still prove insufficient for Complexity to emerge in the cellular array*



Cellular Nonlinear Network



Necessary Conditions for Complexity

Murray Gell-Mann :

- Amplification of fluctuations

Fluctuations
in which physical variable ?

- position ?
- velocity ?
- pressure ?
- temperature ?
- chemical concentration ?
- voltage ?
- current ?

Answer:

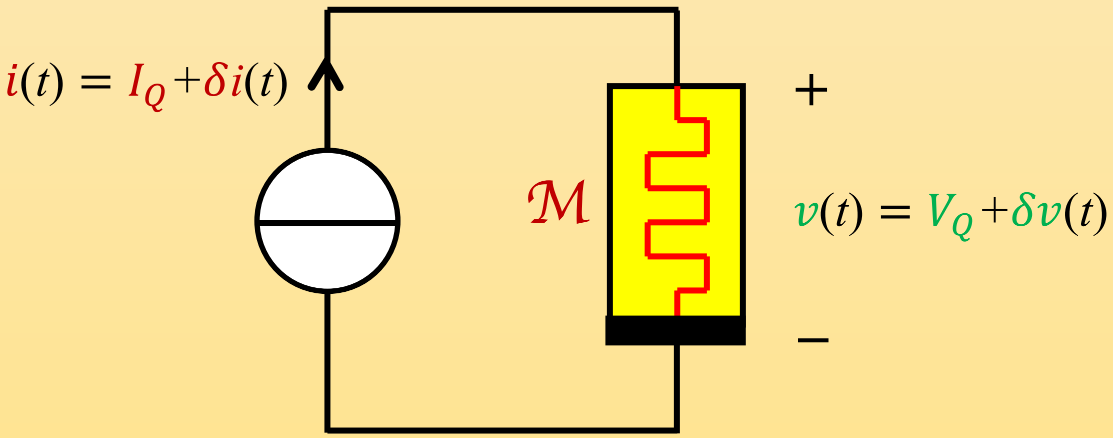
None of the above !

The **local activity principle** asserts that the **correct** physical **variable** whose **amplification** is essential for **complexity** to emerge is ***energy***.

Definition: Local Activity

Any system
is said to be
locally active
iff
it is capable of
amplifying infinitesimal
fluctuations in energy

Locally Active Memristor



A current-controlled memristor

biased at $Q = (V_Q, I_Q)$

Definition:

A current-controlled memristor $\mathcal{M}$ is locally active at an Operating Point $Q = (V_Q, I_Q)$ if there exists an admissible small-signal stimulus $\delta i(t)$ such that

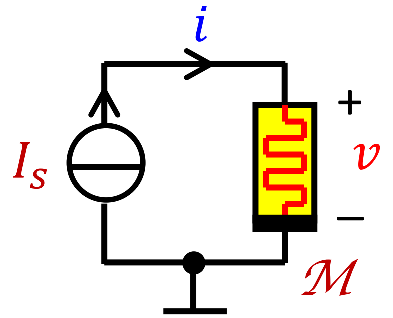
$$\delta \mathcal{E}(t_0, \bar{t}) = \int_{t_0}^{\bar{t}} \delta v(\tau) \cdot \delta i(\tau)\, d\tau < 0$$

for some finite time $\bar{t}$

- *Impractical for Testing!*
- *Luckily, there exists a powerful theorem which simplifies this investigation*
- *Let us first introduce relevant preliminaries*

Recipe for the derivation of the DC voltage-current locus of a memristor

- First-oder current-controlled generic memristor model: $\begin{cases} \dot{x} = f(x, i) \\ v = R(x) \cdot i \end{cases}$



- Apply a DC current source $i_s = I_s$ across the generic memristor $\Longrightarrow$ $I = I_s$

$\Longrightarrow$ Solve $\dot{x} = f(x, I_s) = 0$ $\Longrightarrow$ State solutions: $X_1(I_s), \ldots, X_n(I_s)$

$\Downarrow$

Calculate the corresponding voltage values from Ohm‘s law

$V_1(I_s) = R(X_1(I_s)) \cdot I_s, \quad \ldots, \quad V_n(I_s) = R(X_n(I_s)) \cdot I_s$

$\Downarrow$

Mark the following points on the current-voltage plane

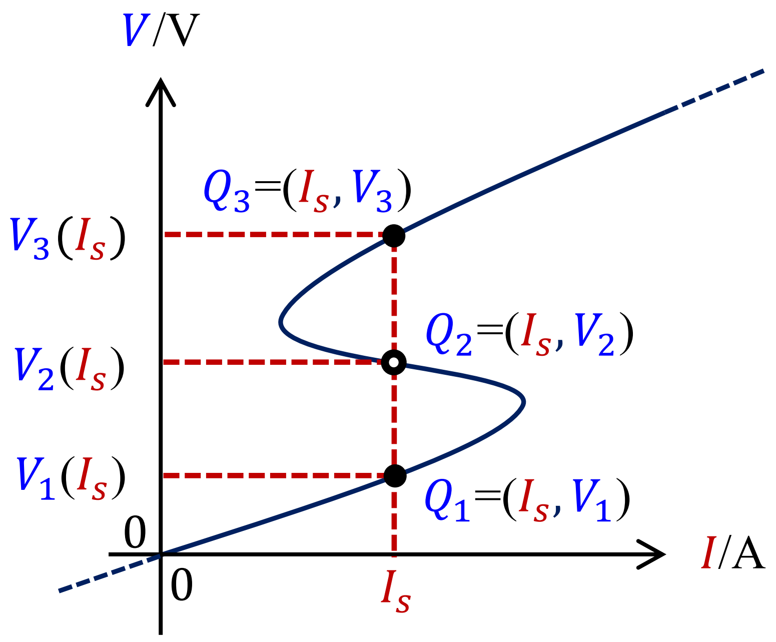
$(I_s, V_1(I_s)), \quad \ldots, \quad (I_s, V_n(I_s))$

$\Downarrow$

Repeat the above procedure for each value of $I_s \in (-\infty, \infty)$

$\Downarrow$

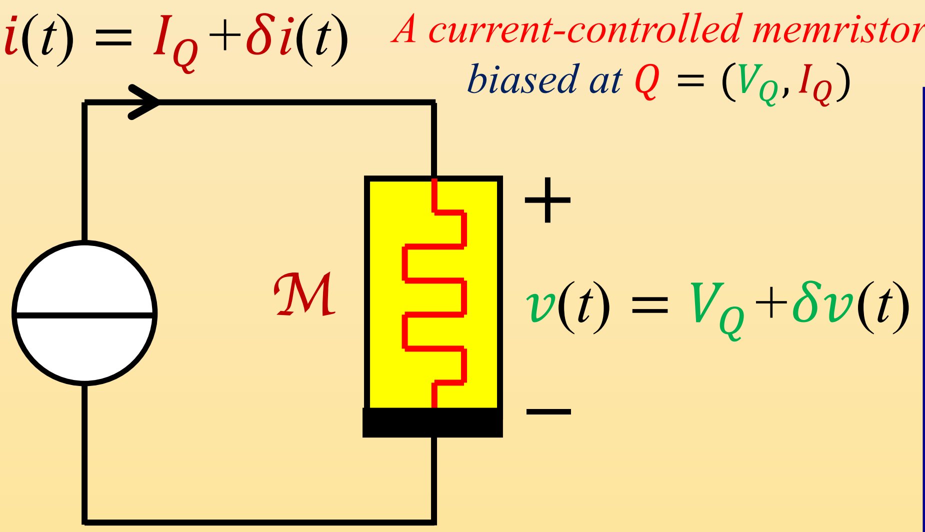
Interpolate the current-voltage pairs derived in all the iterations

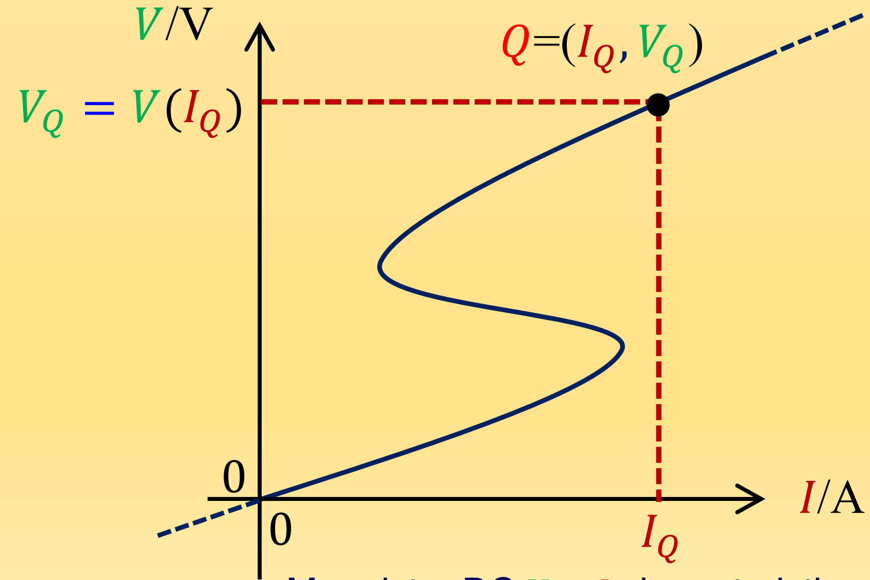


Memristor DC $V - I$ characteristic

Small-signal impedance of a current-driven memristor

A current-controlled memristor biased at $Q = (V_Q, I_Q)$





Memristor DC $V - I$ characteristic

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, i) \\ v = R(\boldsymbol{x}, i) \cdot i \ \text{ with } \lim_{i \to 0\text{A}} R(\boldsymbol{x}, i) \neq \infty \end{cases}$$

DAE set of a n^{th}-order extended memristor, $\boldsymbol{x} = [x_1, x_2, \dots, x_n]^{\text{T}} \in \mathbb{R}^n$

Steps:

1. *Linearize the DAE set about an operating point $Q = (I_Q, V_Q)$*
2. *Transform the linearized system in the Laplace domain*
3. *The local impedance of the memristor is computed via*

$$Z_Q(s) \triangleq \frac{\mathcal{L}\{\delta v(t)\}}{\mathcal{L}\{\delta i(t)\}} = \frac{a_0 + a_1 \cdot s + a_2 \cdot s^2 + \cdots + a_m \cdot s^m}{b_0 + b_1 \cdot s + b_2 \cdot s^2 + \cdots + b_n \cdot s^n}$$

with $m \leq n$.

Local Activity Theorem

A **current-driven one-port** is ***Locally Active*** **at *Q* ⇔ any one of 4 conditions applies:**

1. $Z_Q(s)$ has a pole in $Re\{s\} > 0$

2. $Z_Q(s)$ has a simple pole $s = j\omega_P$ on the imaginary axis, and

$$\mathrm{Res}(Z_Q, j\omega_P) \triangleq \lim_{s \to j\omega_P} (s - j\omega_P)\, Z_Q(s)$$

is either a complex number or a negative real number

3. $Z_Q(s)$ has a multiple pole $s = j\omega_P$ on the imaginary axis

4. $\Re\{Z_Q(j\omega)\} < 0$ for at least one real-valued $\omega = \omega_0$

- ***Note: conditions 2. and 3. refer to marginal cases***

Definition: Edge of Chaos

A one-port

is said to be on the

Edge of Chaos

if

it is locally active at some

asymptotically-stable

operating point Q

(only condition 4. from Local Activity Theorem applies)

A **current-driven one-port** is ***Locally Active*** **at *Q* ⇔ any one of 4 conditions applies:**

1. $Z_Q(s)$ has a pole in $Re\{s\} > 0$

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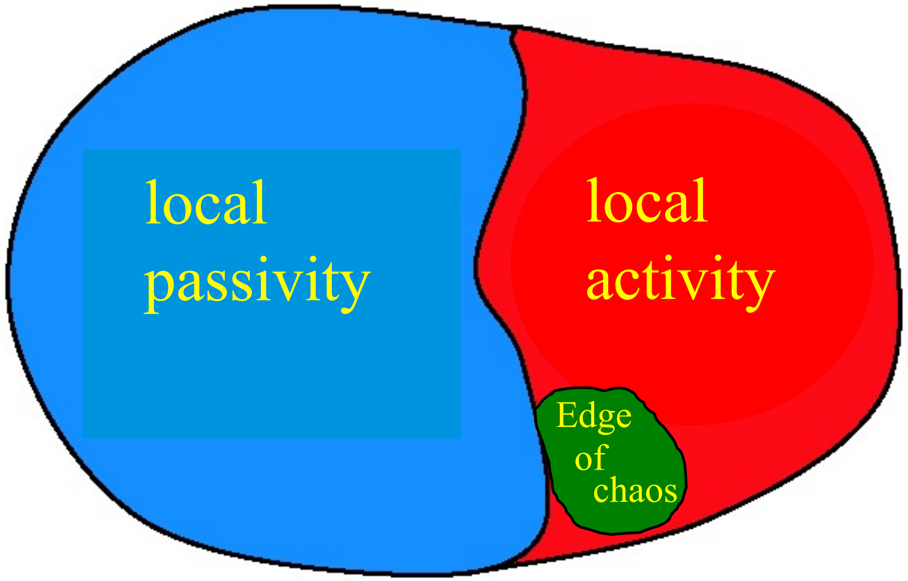
4. $\Re\{Z_Q(j\omega)\} < 0$ for at least one real-valued $\omega = \omega_0$



Edge of Chaos
is the
"Pearl"
Embedded within
the domain of
Local Activity

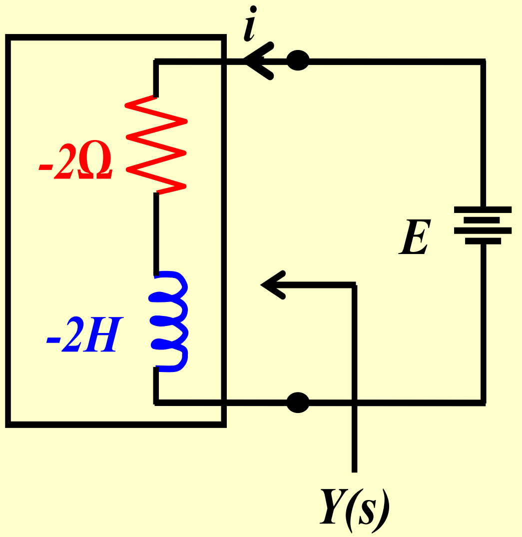
Edge of Chaos

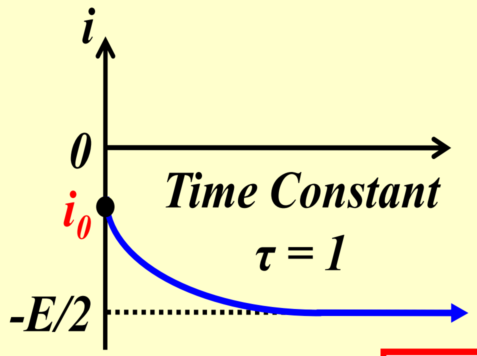
is an *innate* Characteristic of a ***dynamical system.*** It *does not depend on* the *external environment* it interacts with.

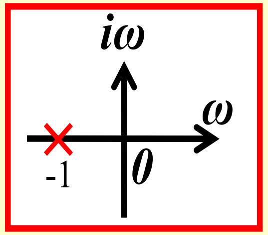
Chua's

Riddle:

Solution



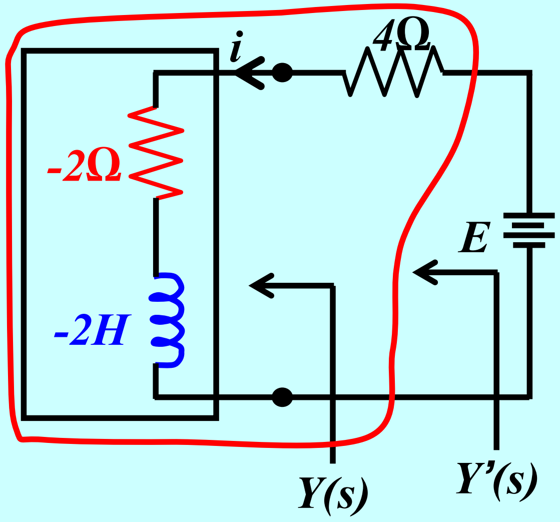


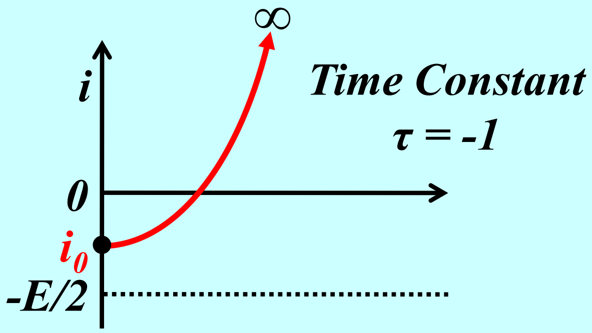


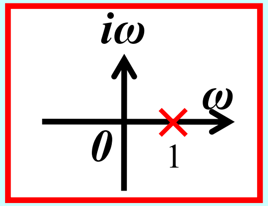
Admittance, $Y(s)=\frac{I(s)}{V(s)}=\frac{1}{(-2-2s)}=\frac{-1}{2(s+1)}$

$$Y(i\omega)=\frac{-1}{2(1+i\omega)}=\frac{-(1-i\omega)}{2(1+i\omega)(1-i\omega)}=\frac{-1}{2(1+\omega^2)}+i\frac{\omega}{2(1+\omega^2)}$$

$$Re\ Y(i\omega)=\frac{-1}{2(1+\omega^2)}<0,\quad -\infty<\omega<\infty$$







Admittance, $Y'(s)=\frac{I(s)}{V(s)}=\frac{1}{(2-2s)}=\frac{-1}{2(s-1)}$

$$Y'(i\omega)=\frac{-1}{2(-1+i\omega)}=\frac{-1(-1-i\omega)}{2(-1+i\omega)(-1-i\omega)}=\frac{1}{2(1+\omega^2)}+i\frac{\omega}{2(1+\omega^2)}$$

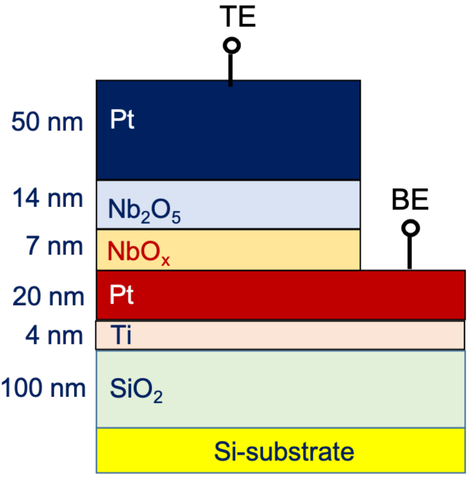
$$Re\ Y'(i\omega)=\frac{1}{2(1+\omega^2)}>0,\quad -\infty<\omega<\infty$$

The voltage-controlled one-port within the black box is poised on the Stable Locally-Active operating regime, also referred to as Edge of Chaos

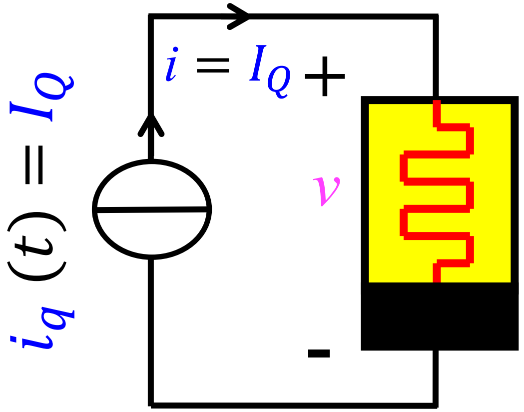
Including the passive and linear resistor in series with the original one-port, the resulting overall voltage-controlled one-port within the red box is poised on the Unstable Locally-Active operating regime

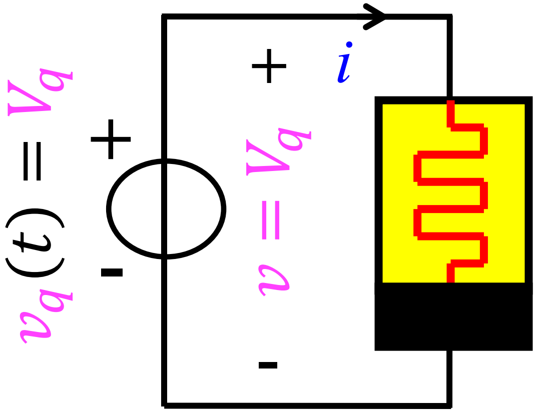
A miniaturized volatile niobium oxide memristor with locally-active behaviour

Experimental observations

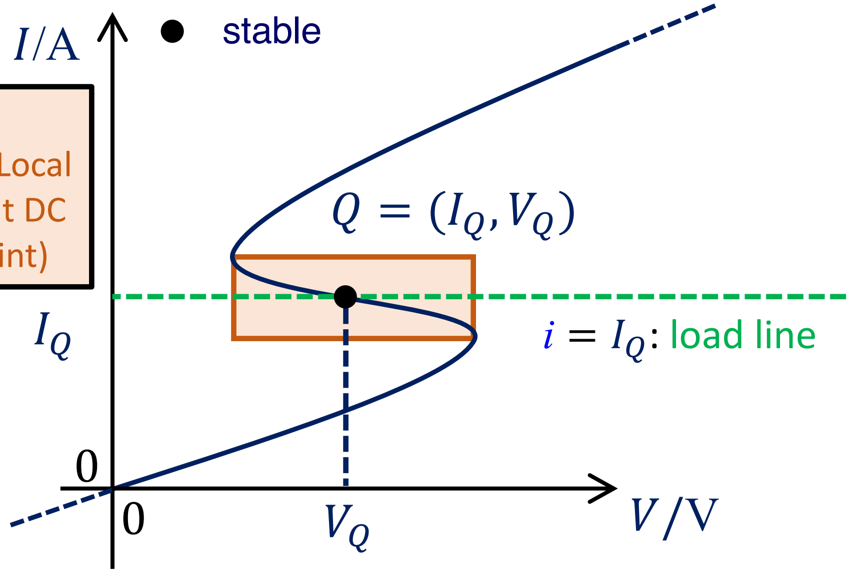


NbO device stack from NaMLab

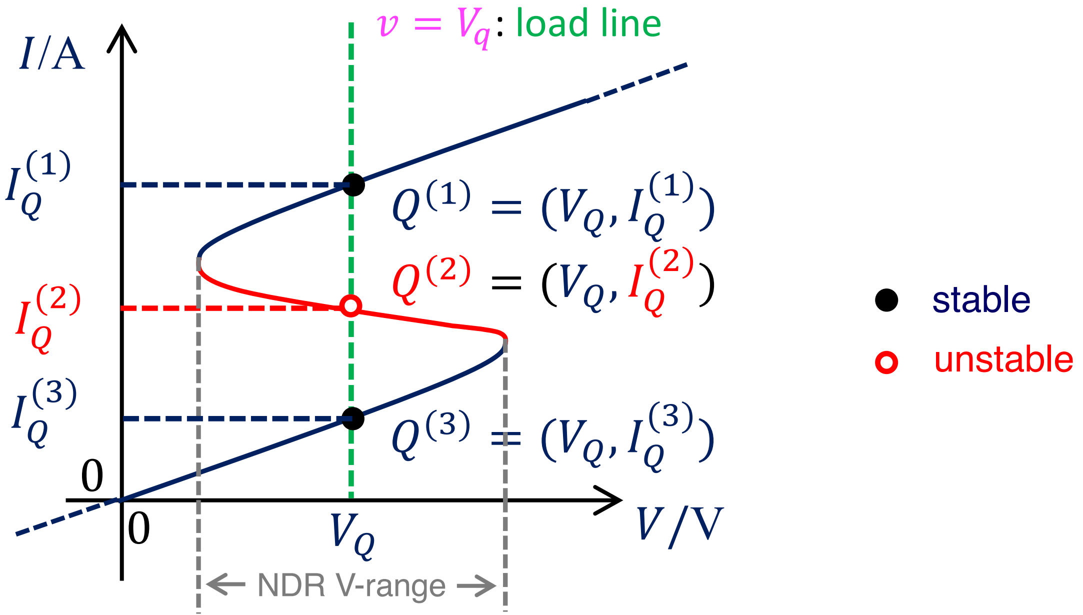




NDR region
(condition 4. from the Local Activity Theorem holds at DC at any NDR operating point)



Qualitative sketch of the device DC $V - I$ characteristic obtained under current sweep. **Blue: stable branch**



Qualitative sketch of the device DC $I - V$ characteristic obtained under voltage sweep. Blue: stable branch. Red: unstable branch

A generic memristor model for the NbO threshold switch from NaMLab

- To explain the experimental measurements, we developed a DAE set-based model for the NaMLab memristor:

$$\begin{cases} \frac{dx}{dt} = g(x, v) \\ i = G(x)v \end{cases}$$

with state evolution and memductance functions respectively expressed by

$$g(x, v) = a_0 + a_1 x + b_2 v^2 + c_{21} v^2 x + c_{22} v^2 x^2 + c_{23} v^2 x^3 + c_{24} v^2 x^4 + c_{25} v^2 x^5$$

and

$$G(x) = d_0 + d_1 x + d_2 x^2 + d_3 x^3 + d_4 x^4$$

Table 1 Parameter setting

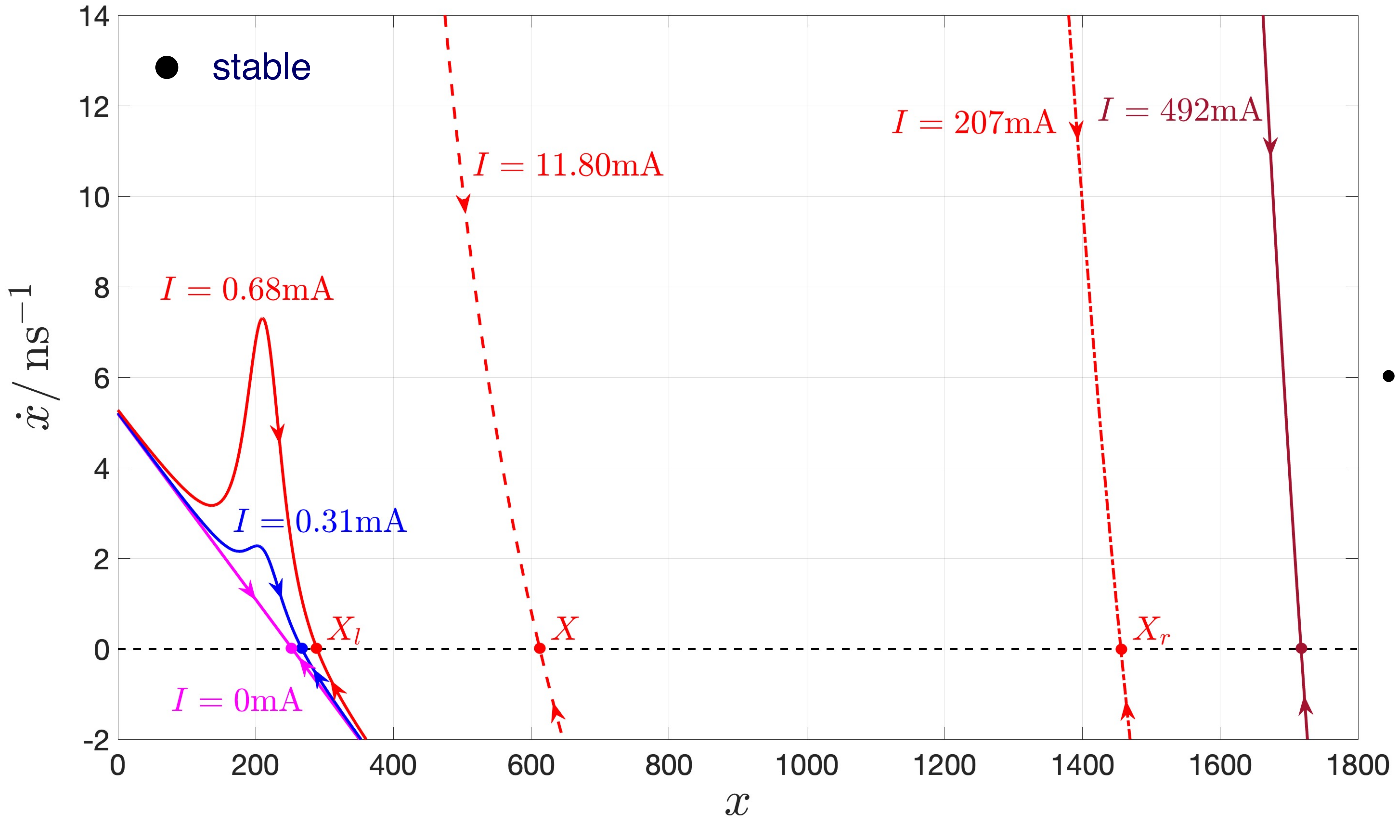
a_0	a_1	b_2	c_{21}	c_{22}	c_{23}	c_{24}
$5.19 \cdot 10^9$	$-2.05 \cdot 10^7$	$7.21 \cdot 10^9$	$-0.07 \cdot 10^9$	$2.27 \cdot 10^5$	$-2.40 \cdot 10^2$	$1.25 \cdot 10^{-1}$

c_{25}	d_0	d_1	d_2	d_3	d_4
$-2.69 \cdot 10^{-5}$	$6.50 \cdot 10^{-3}$	$-6.66 \cdot 10^{-5}$	$2.14 \cdot 10^{-7}$	$-2.14 \cdot 10^{-10}$	$1.19 \cdot 10^{-13}$

Memristor Dynamic Route Map (DRM) under Current Control

Rewrite the DAE set as

$$\begin{cases} \dfrac{dx}{dt} = g(x, G^{-1}(x)i) \\ v = G^{-1}(x)i \end{cases}$$



Memristor DRM under a range of DC current values

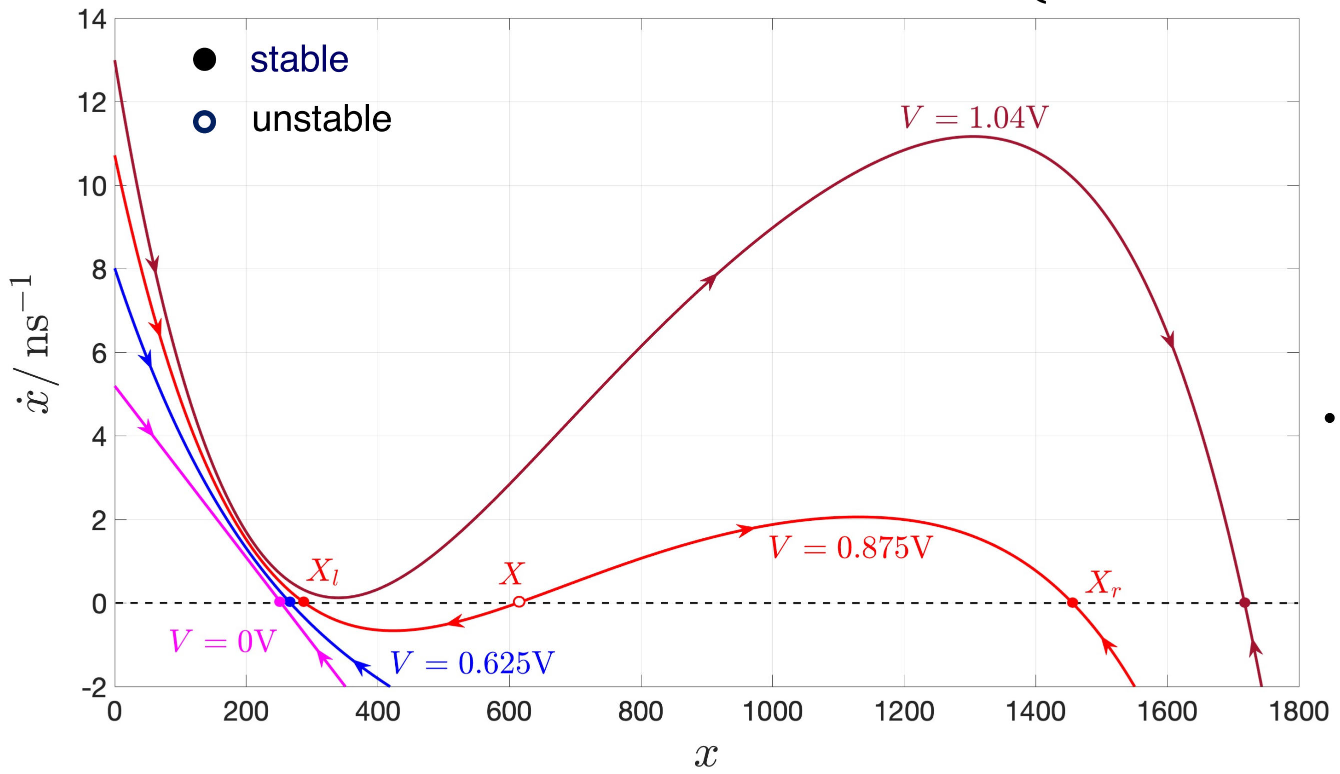
- Irrespective of the DC current, there exists <u>one and only one globally asymptotically stable operating point</u> [1]

[1] A. Ascoli, S. Slesazeck, H. Mähne, R. Tetzlaff, and T. Mikolajick, "Nonlinear dynamics of a locally-active memristor," *IEEE TCAS–I*, vol. 62, no. 4, pp. 1165–1175, 2015

Memristor Dynamic Route Map (DRM) under Voltage Control

Memristor DAE set:

$$\begin{cases} \dfrac{dx}{dt} = g(x, v) \\ i = G(x)v \end{cases}$$

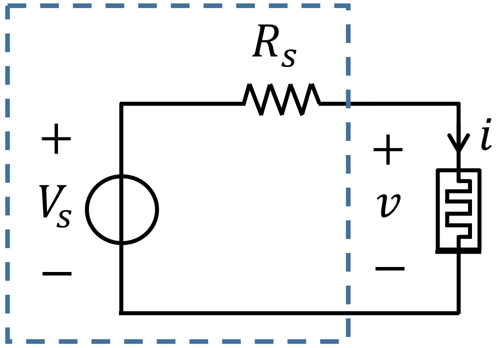


Memristor DRM under a range of DC voltage values

- If the DC voltage lies within the NDR region, there exists <u>three possible operating points</u>, of which <u>the intermediate NDR one is unstable</u> [1]

[1] A. Ascoli, S. Slesazeck, H. Mähne, R. Tetzlaff, and T. Mikolajick, “Nonlinear dynamics of a locally-active memristor,” *IEEE TCAS–I*, vol. 62, no. 4, pp. 1165–1175, 2015

Biasing circuit for stabilizing a NDR operating point on the DC locus of the voltage-controlled device



DC biasing circuit [1]

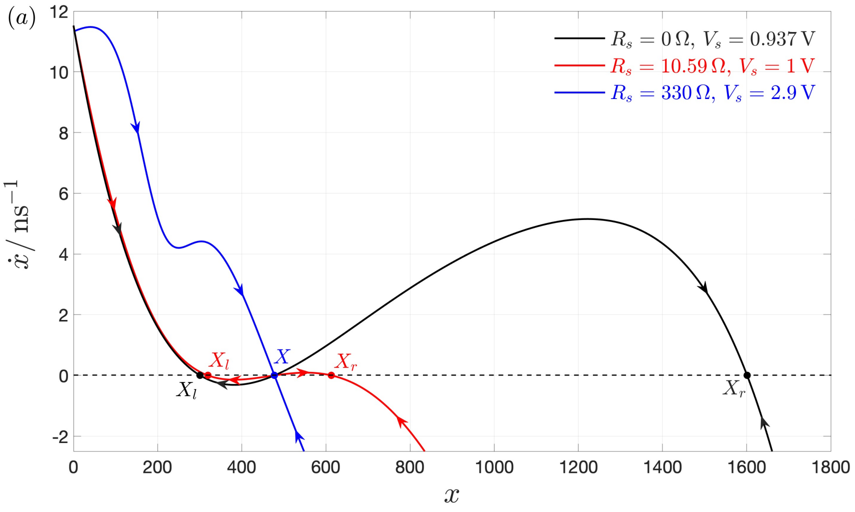
Note:

$$v = \frac{V_S}{1 + G(x) \cdot R_S}$$ memristor voltage

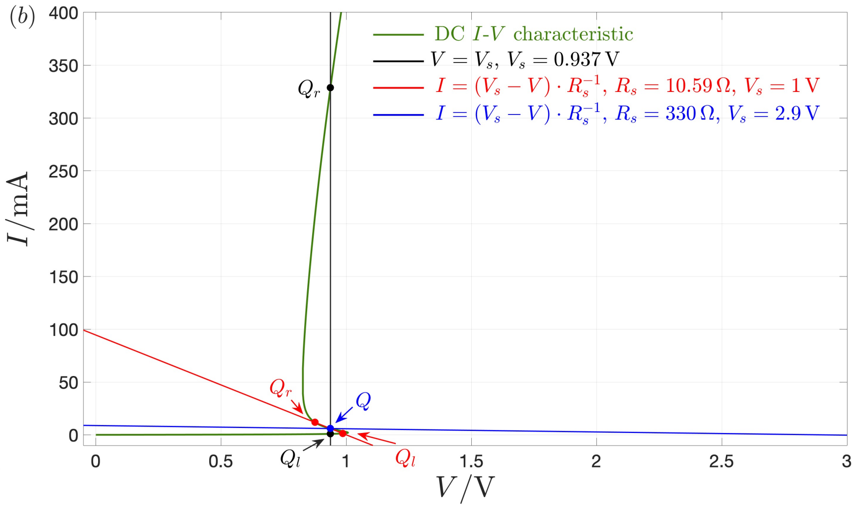
$$i = \frac{V_S - v}{R_S}$$ load line

[1] A. Ascoli, S. Slesazeck, H. Mähne, R. Tetzlaff, and T. Mikolajick, "Nonlinear dynamics of a locally-active memristor," *IEEE TCAS–I*, vol. 62, no. 4, pp. 1165–1175, 2015

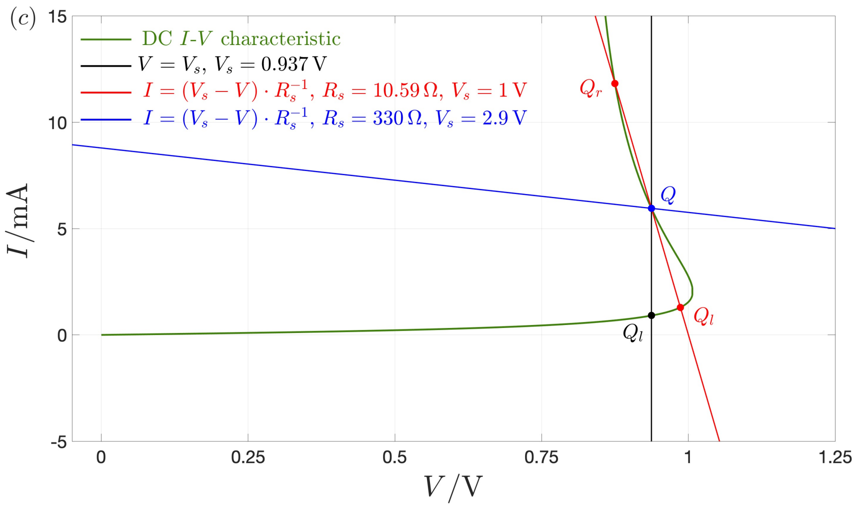
Stabilization of an Operating Point on the NDR Region of the DC Characteristic of the Memristor under Voltage Control



Triplet of memristor State Dynamic Routes (SDRs) sharing one NDR operating point



Memristor DC I-V locus and triplet of Load Lines corresponding to the SDRs from (a)



Enlarged view of (b) in the low current regime

- Applying a DC voltage V_S across the one resistor R_S-one memristor $\mathcal{M}$ series one-port in such a way that the load line intersects the memristor DC characteristic in some NDR operating point Q, the latter is stabilised provided

$$R_S > -r\Big|_Q \triangleq -\frac{1}{\left.\frac{di}{dv}\right|_Q}$$

stabilization condition of a NDR operating point Q

i.e. if and only if the modulus of the slope of the load line, i.e. $1/R_S$, is smaller than the modulus of the slope of the device DC characteristic at Q, i.e. $-1/r|_Q$, which is the modulus of the memristor negative differential conductance at Q

Small-Signal Equivalent Circuit Model of the Threshold Switch

- Memristor DAE set:

$$\begin{cases} \dfrac{dx}{dt} = g(x, v) \\ i = i(x, v) = G(x)v \end{cases}$$

where

$$g(x, v) = a_0 + a_1 x + b_2 v^2 + c_{21} v^2 x + c_{22} v^2 x^2 + c_{23} v^2 x^3 + c_{24} v^2 x^4 + c_{25} v^2 x^5$$

$$G(x) = d_0 + d_1 x + d_2 x^2 + d_3 x^3 + d_4 x^4$$

- Local admittance of the voltage-controlled memristor about an operating point $Q = (V, I)$:

$$Y_Q(s) = \frac{\mathcal{L}\{\delta i(t)\}}{\mathcal{L}\{\delta v(t)\}} = K \cdot \frac{s - z_{Y_Q}}{s - p_{Y_Q}}$$

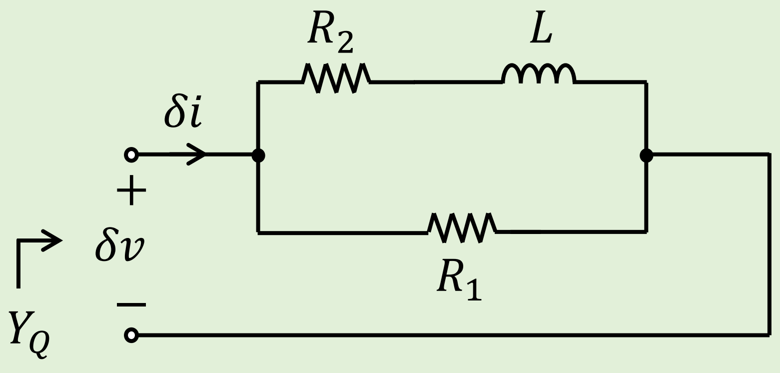
where

$$\begin{cases} K = d \\ z_{Y_Q} = \dfrac{a \cdot d - b \cdot c}{d} \\ p_{Y_Q} = a \end{cases}$$

and

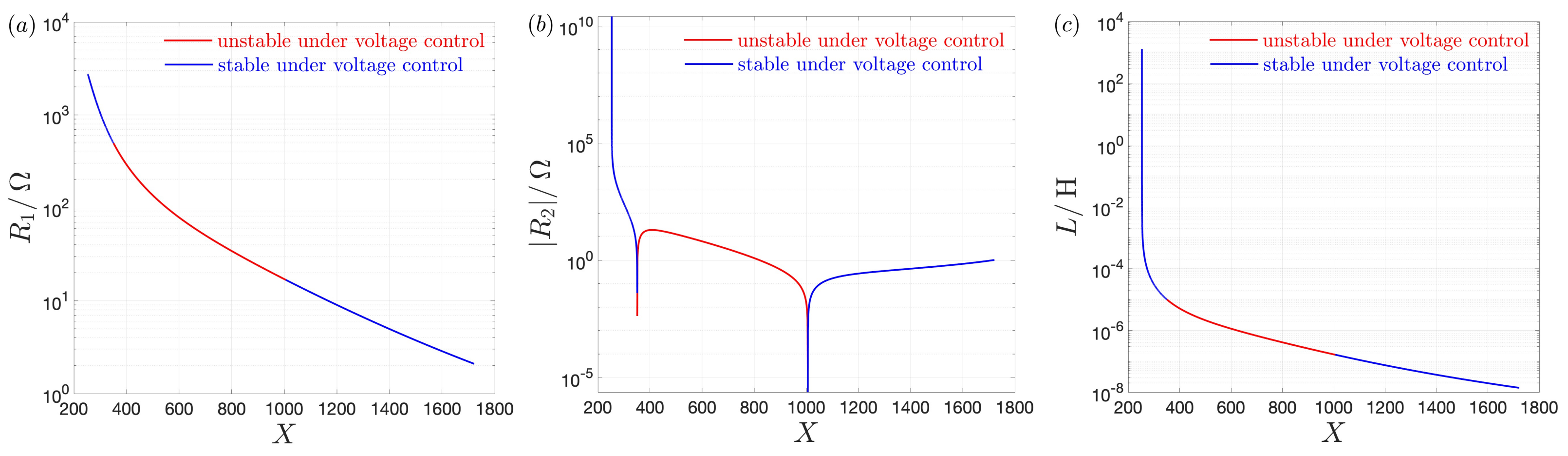
$$a \triangleq \left.\frac{\partial g(x, v)}{\partial x}\right|_Q \quad b \triangleq \left.\frac{\partial g(x, v)}{\partial v}\right|_Q$$

$$c \triangleq \left.\frac{\partial i(x, v)}{\partial x}\right|_Q \quad d \triangleq \left.\frac{\partial i(x, v)}{\partial v}\right|_Q$$



$$R_1 = \frac{1}{K}, \quad R_2 = -\frac{p_{Y_Q}}{K \cdot (p_{Y_Q} - z_{Y_Q})}, \quad L = \frac{1}{K \cdot (p_{Y_Q} - z_{Y_Q})}$$

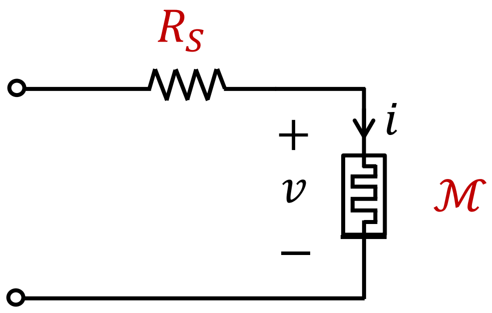
Memristor small-signal equivalent circuit about Q



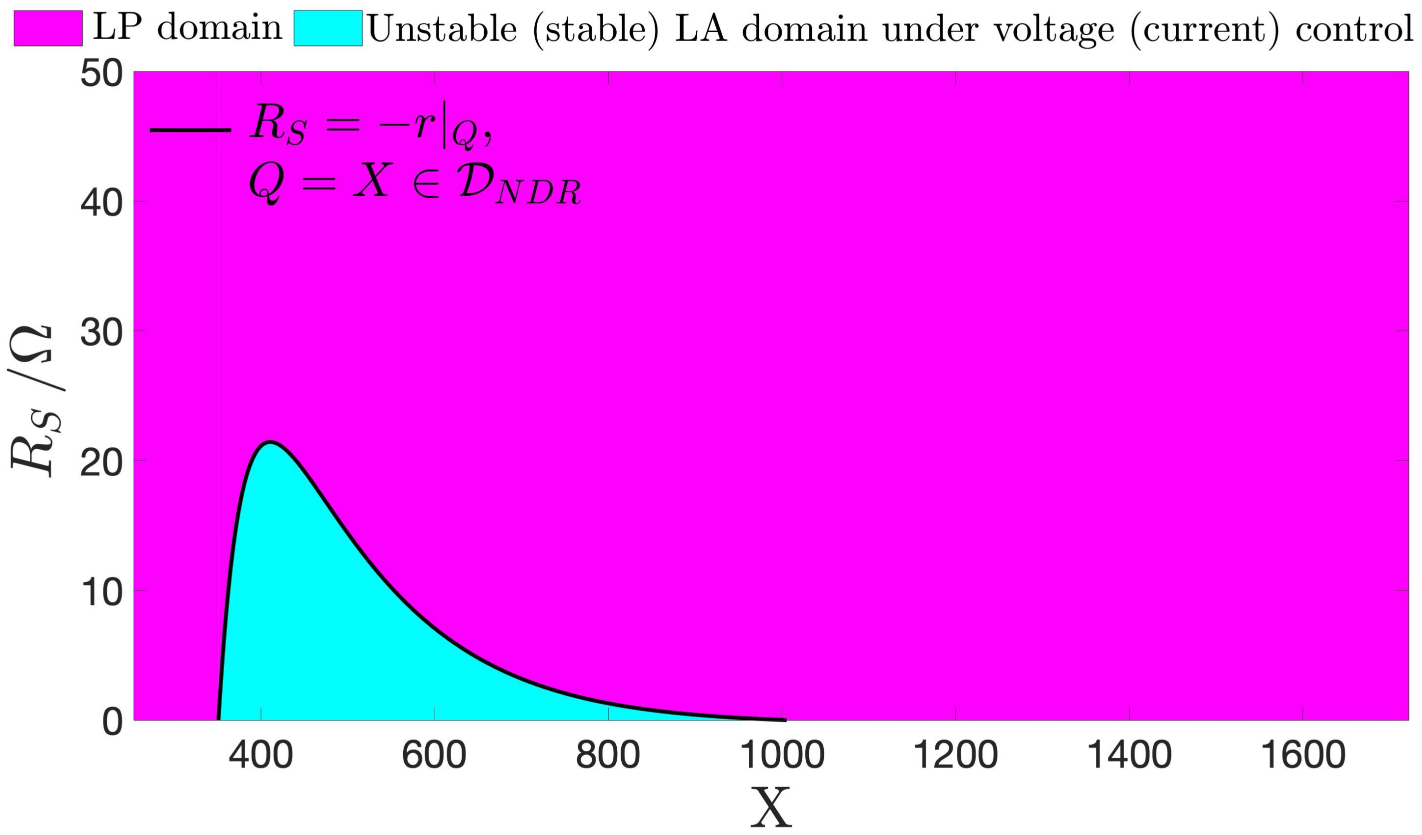
Dependence of the memristor small-signal equivalent circuit parameters upon its DC operating point [2]

[2] A. Ascoli, A.S. Demirkol, R. Tetzlaff, S. Slesazeck, T. Mikolajick, and L.O. Chua, “On Local Activity and Edge of Chaos in a NaMLab Memristor”, *Frontiers in Neuroscience*, 2021, DOI: 10.3389/fnins.2021.651452

Classification of the Possible Operating Regimes of the R_S-$\mathcal{M}$ one-port



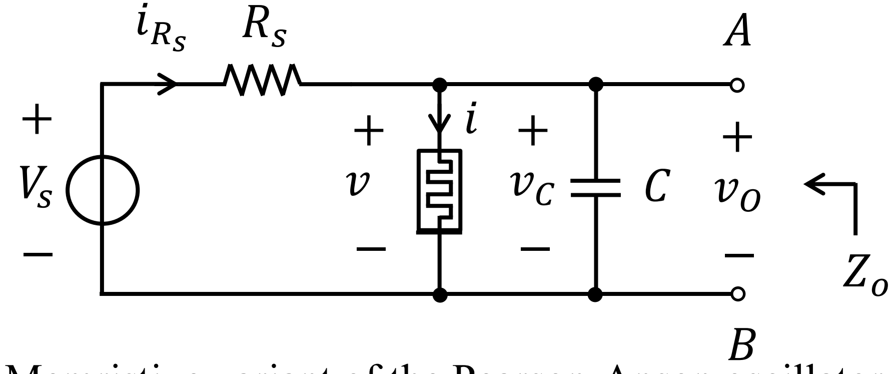
A
R_S–$\mathcal{M}$ one–port



Classification of all possible operating regimes of the R_S-$\mathcal{M}$ one-port under both voltage and current control

$$R_S = -r\Big|_Q \triangleq -\frac{1}{\left.\frac{di}{dv}\right|_Q}$$: locus of points along the frontier between LP and LA operating regimes

Memristive Variant of the Pearson-Anson Relaxation Oscillator and Its Small-Signal Equivalent Circuit Model



Memristive variant of the Pearson-Anson oscillator

State equations of the second-order cell:

$$\begin{cases} \dfrac{dx}{dt} = g(x, v) \\ \dfrac{dv}{dt} = \dfrac{1}{C}\left(\dfrac{V_s - v}{R_s} - i\right) \end{cases}$$

where

$$i = G(x) \cdot v$$

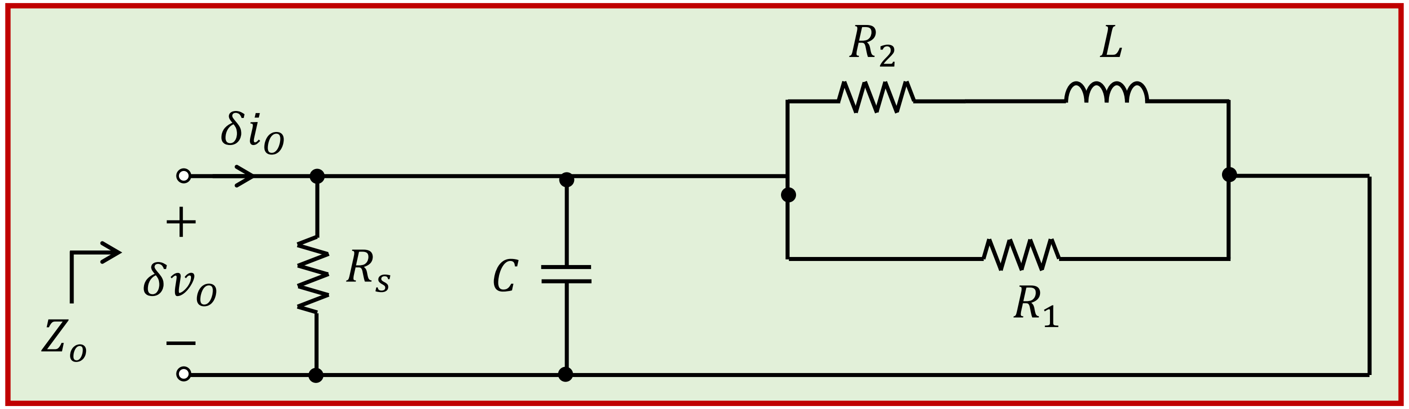
- Local input impedance of the oscillator at the coupling port $A - B$

$$Z_o(s) = \frac{\mathcal{L}\{\delta v_O(t)\}}{\mathcal{L}\{\delta i_O(t)\}} = K \cdot \frac{s - s_{z,Z_o}}{(s - s_{p_1,Z_o}) \cdot (s - s_{p_2,Z_o})}$$

where K, s_{z,Z_o}, and s_{p_i,Z_o} for $i \in \{1,2\}$ may be expressed in terms of the parameters of the memristor small-signal equivalent circuit model as

$$K = \frac{1}{C} \quad , \quad s_{z,Z_o} = -\frac{R_2}{L} \quad ,$$

$$s_{p_i,Z_o} = -\left(\frac{R_2}{L} + \frac{R_1 + R_s}{C \cdot R_1 \cdot R_s}\right) \pm \frac{1}{2} \cdot \sqrt{\left(\frac{R_2}{L} + \frac{R_1 + R_s}{C \cdot R_1 \cdot R_s}\right)^2 - 4 \cdot \frac{1}{L \cdot C}\left(1 + R_2 \cdot \frac{R_1 + R_s}{R_1 \cdot R_s}\right)}$$

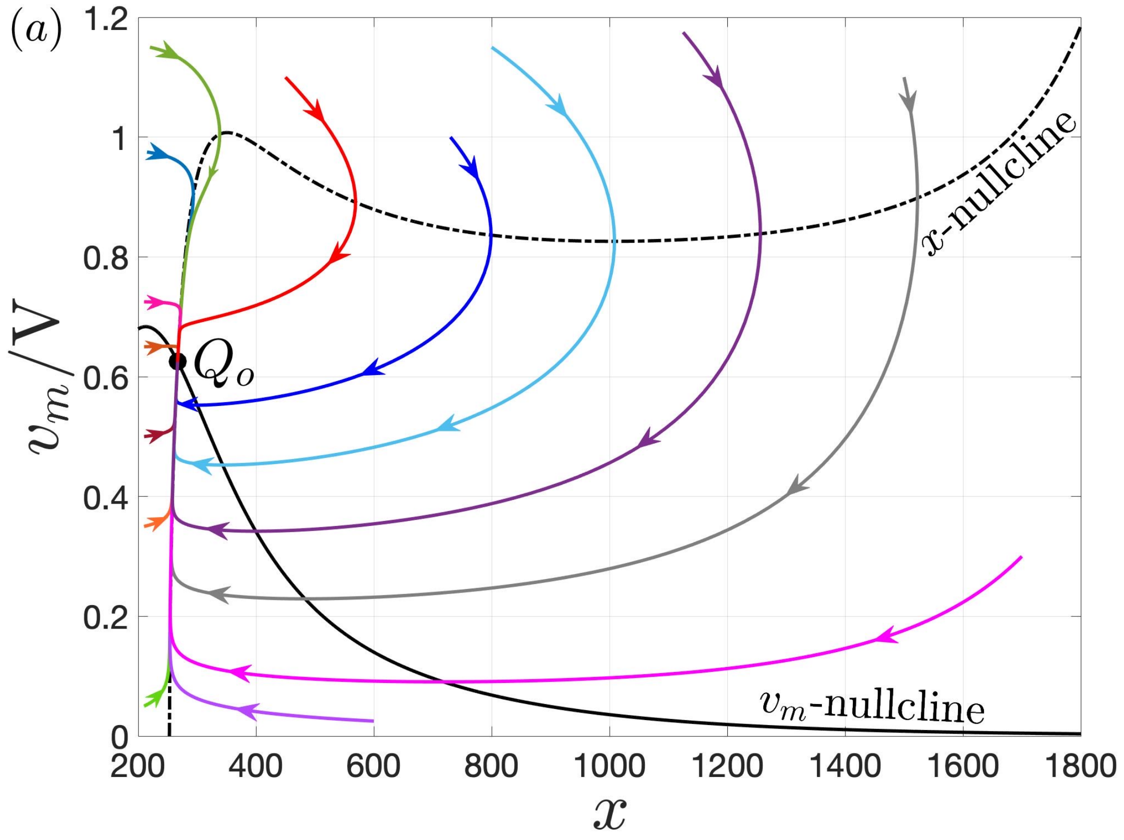


Oscillator small-signal equivalent circuit about $Q_o = (X, V)$ [3]

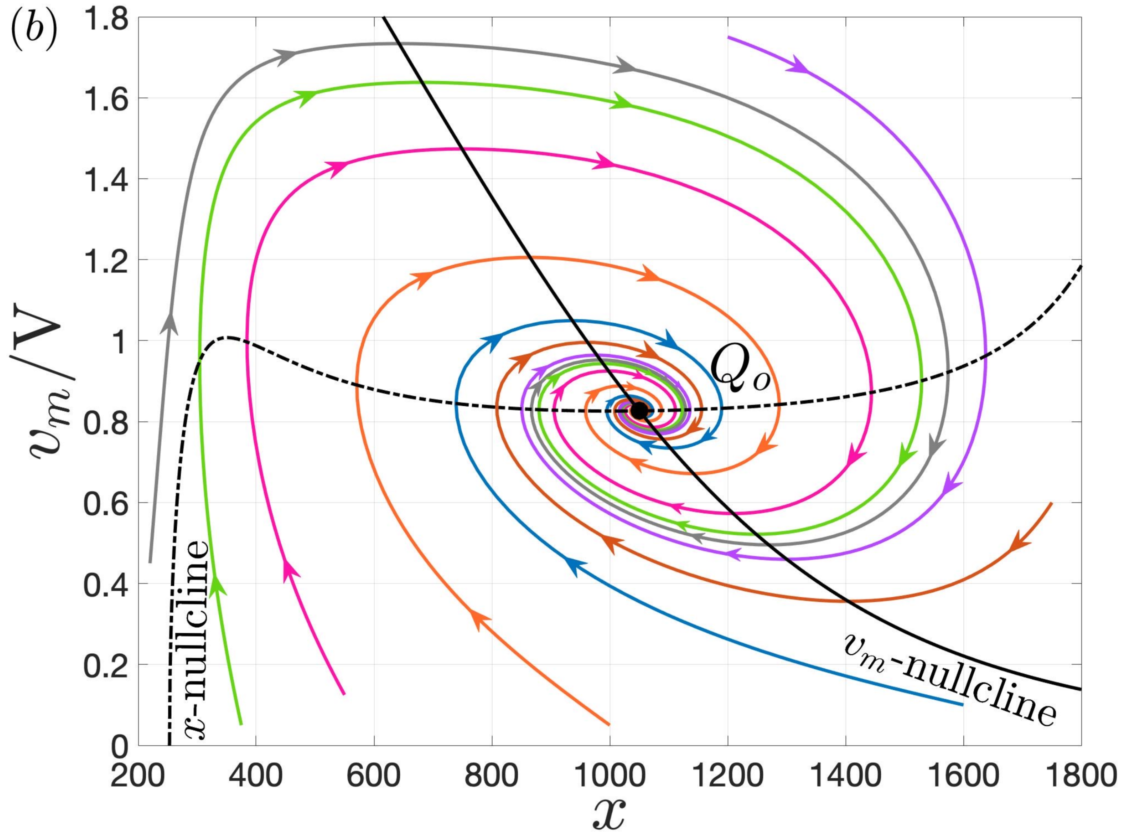
[3] A. Ascoli, A.S. Demirkol, R. Tetzlaff, and L.O.Chua, “Edge of Chaos behind Diffusion-Driven Instabilities in a Two-Memristor-Cell Array,” *IEEE TCAS–I*, 2021, in press

Scenario 1: Cell dynamics when the memristor $\mathcal{M}$ is polarized in one and only one positive differential resistance (PDR) operating point

- If V_S and R_S are such that the load line meets the memristor DC I-V locus at one point $Q = (V, I)$, lying either in the lower or in the upper PDR branch, the cell features one globally asymptotically stable (GAS) operating point $Q_o = (X, V)$, irrespective of C.
- In both simulation results to follow, C was set to 5 nF



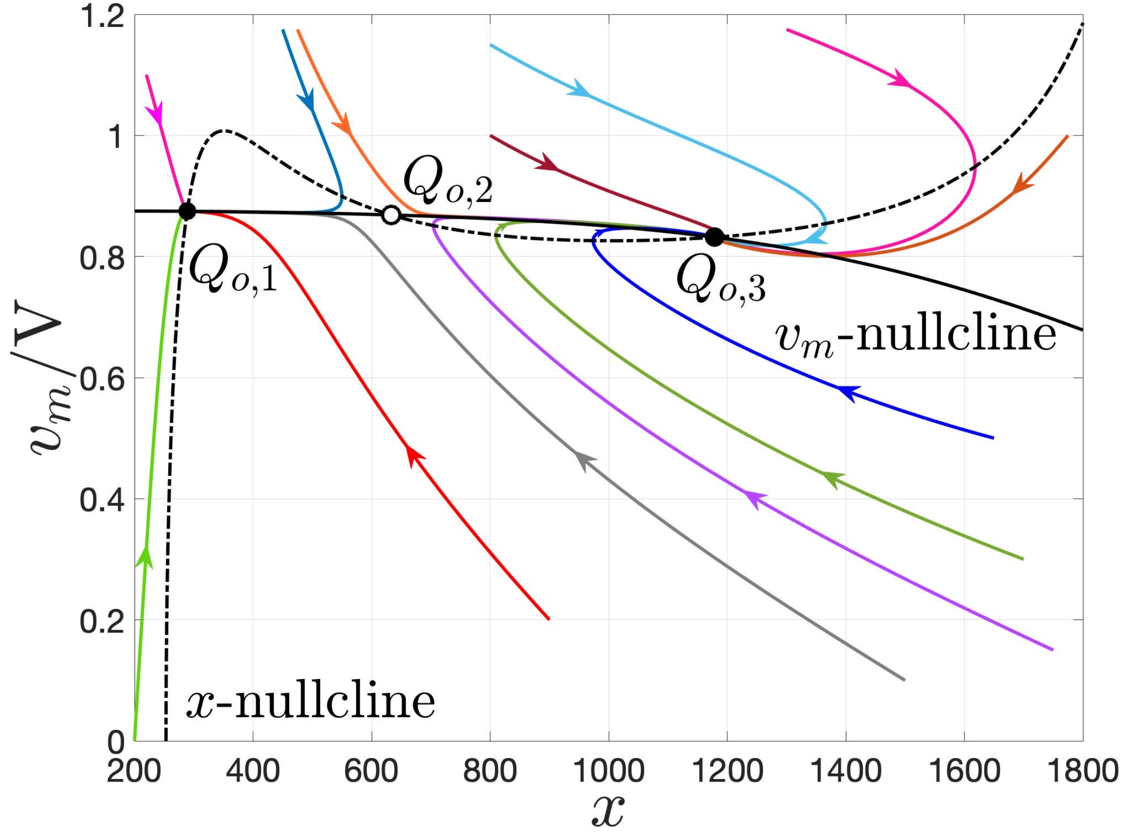
Cell phase portrait for $V_S = 0.7275$ V and $R_S = 330\ \Omega$.



Cell phase portrait for $V_S = 2.5392$ V and $R_S = 30\ \Omega$.

Scenario 2: Cell dynamics when the memristor $\mathcal{M}$ may stabilize in either of the two positive differential resistance (PDR) regions

- Assume V_S and R_S are such that the load line intersects the memristor DC I-V locus at $Q_i = (V_i, I_i)$, where $i \in \{1,2,3\}$. Q_i lies in the lower (upper) PDR branch for $i = 1$ (3), while it sits in the NDR branch for $i = 2$. Correspondingly, the i^{th} cell operating point $Q_{o,i} = (X_i, V_i)$ is found to be locally-stable for each i-value in {1,3} (unstable for $i = 2$), irrespective of C.
- In the simulation result to follow, C was set to 100 nF

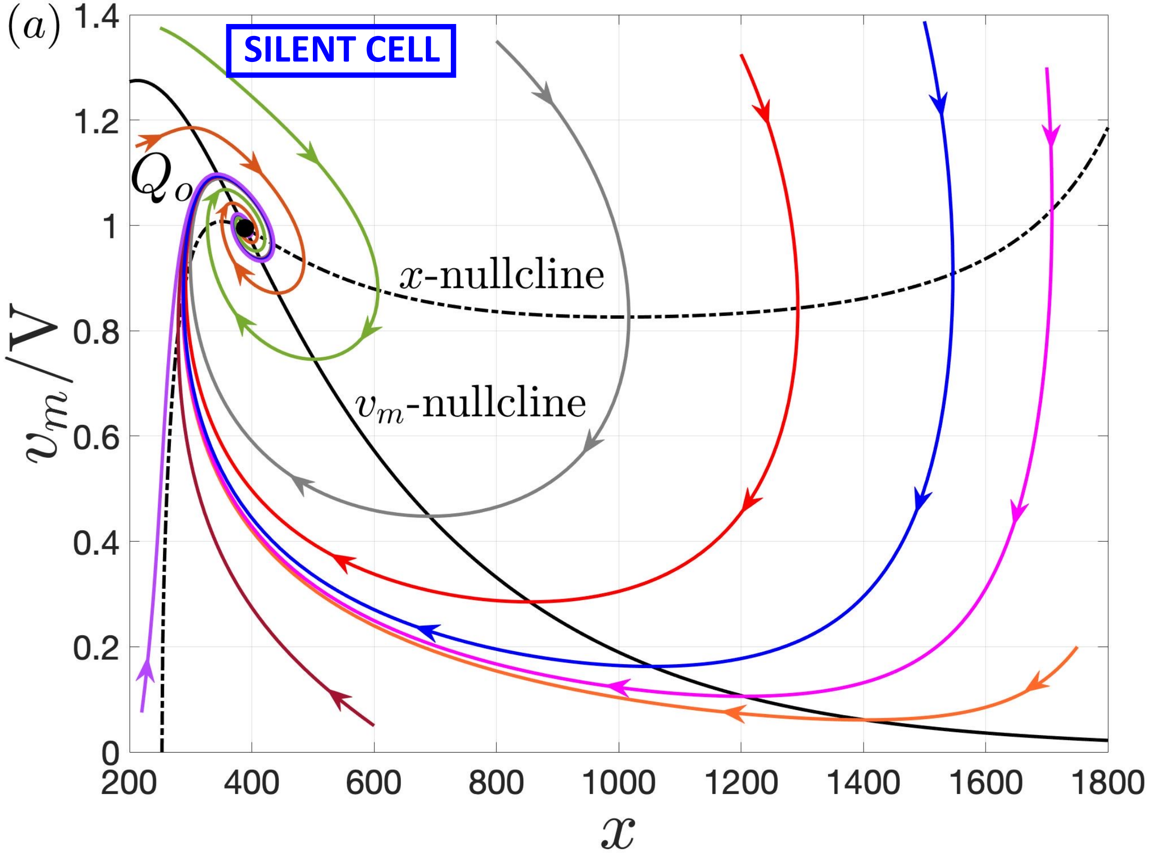


Cell phase portrait for $V_S = 0.875$ V and $R_S = 0.5\ \Omega$.

Scenario 3: Cell dynamics when the memristor $\mathcal{M}$ is polarized in one and only one negative differential resistance (NDR) operating point

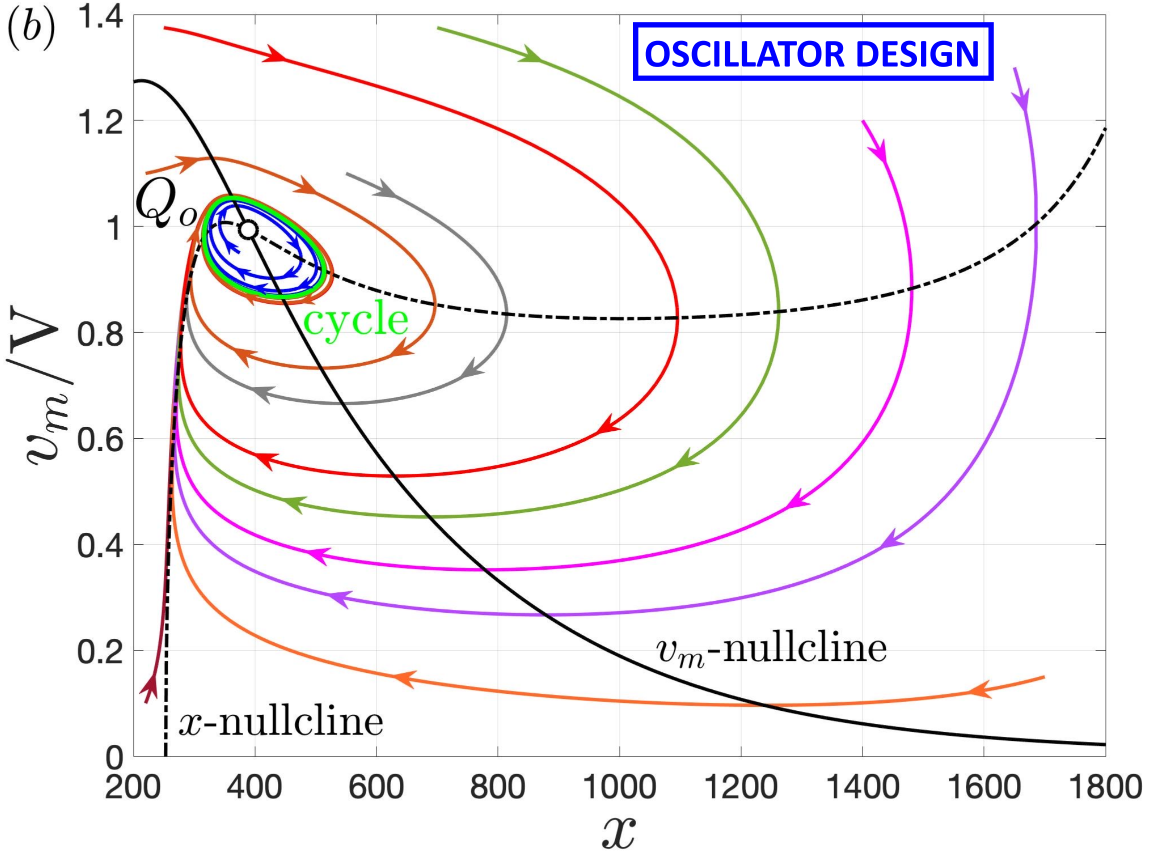
- If V_S and R_S are such that the load line intersects the memristor DC I-V locus only in a point $Q = (V, I)$, lying on the NDR branch, the cell stability in the corresponding operating point $Q_o = (X, V)$ depends critically upon C.

The cell is poised on the Edge of Chaos at Q_o



Cell phase portrait for $V_S = 1.3$ V , $R_S = 100\ \Omega$, and C = 2 nF.

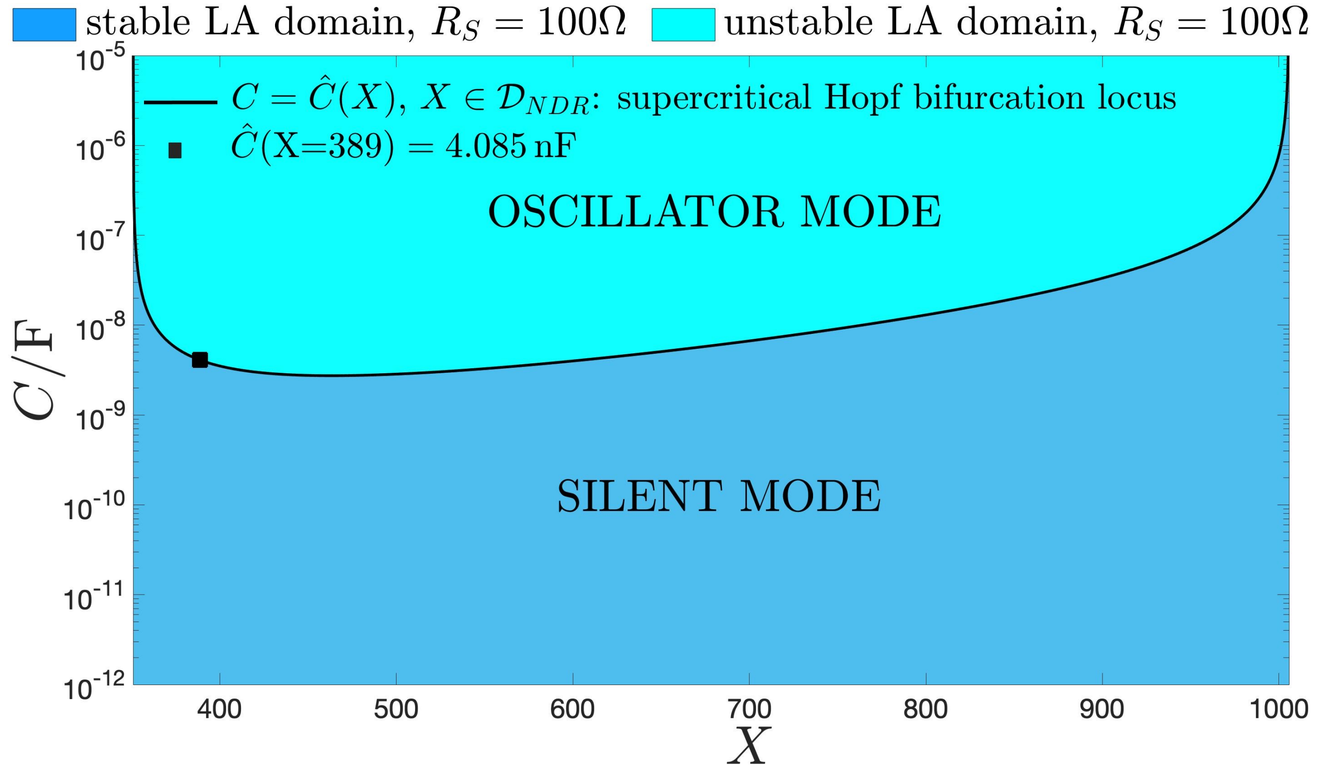
The cell is poised on the Unstable Locally Active Domain at Q_o



Cell phase portrait for $V_S = 1.3$ V , $R_S = 100\ \Omega$ and C = 6 nF .

Classification of Cell Operating Regimes for all Possible Cases Studies in Scenario 3

- Assume the device needs to be polarised in a specific point $Q = (V, I)$ along the NDR branch of its DC V-I locus.
- Let $R_S = 100\Omega$ $\rightarrow$ *the bias point stabilization condition for the voltage-controlled memristor applies throughout the NDR branch*



Classification of all possible operating regimes of the cell in scenario 3

- A supercritical Hopf bifurcation occurs along the locus $C = \hat{C}(X)$.
- Case study: if $V_S = 1.3$V, we have $X = 389$, $V = 0.994$V

For C $< \hat{C} = 4.085$nF the cell is poised on the EOC domain

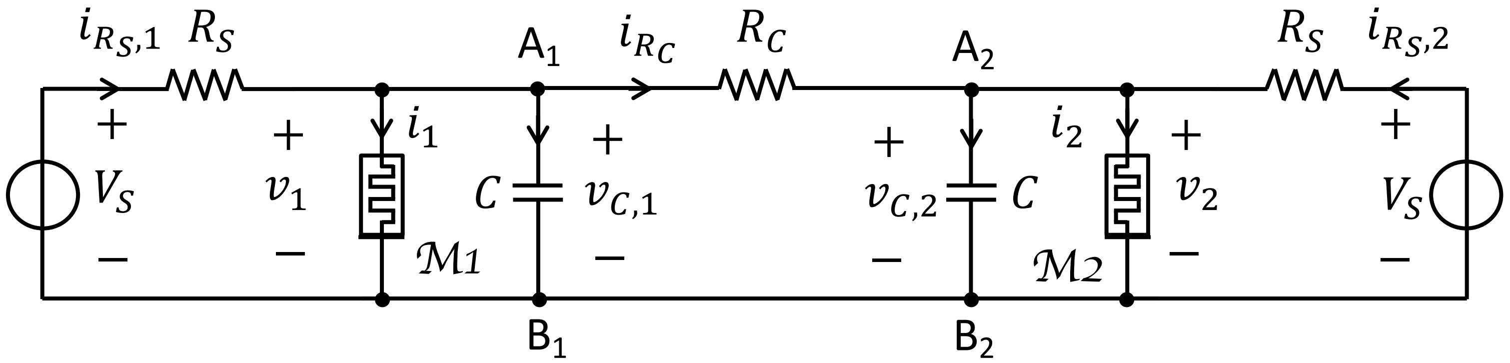
For C $> \hat{C} = 4.085$nF the cell is poised in the unstable LA domain

Fundamental Result

Edge of Chaos Theorem

A stable operating point Q of a given one-port may be destabilized by coupling the one port to a passive environment if and only if
Q is poised on the
Edge of Chaos

Coupled System



- Two identical cells, poised on the EOC on their own, are diffusively coupled through a passive and linear resistor R_C
- State equations:

$$\frac{dx_1}{dt} = g(x_1, v_1)$$

$$\frac{dv_1}{dt} = \frac{1}{C}\left(\frac{V_s - v_1}{R_s} - i_1 + \frac{v_2 - v_1}{R_C}\right)$$

$$\frac{dx_2}{dt} = g(x_2, v_2)$$

$$\frac{dv_2}{dt} = \frac{1}{C}\left(\frac{V_s - v_2}{R_s} - i_2 + \frac{v_1 - v_2}{R_C}\right)$$

where

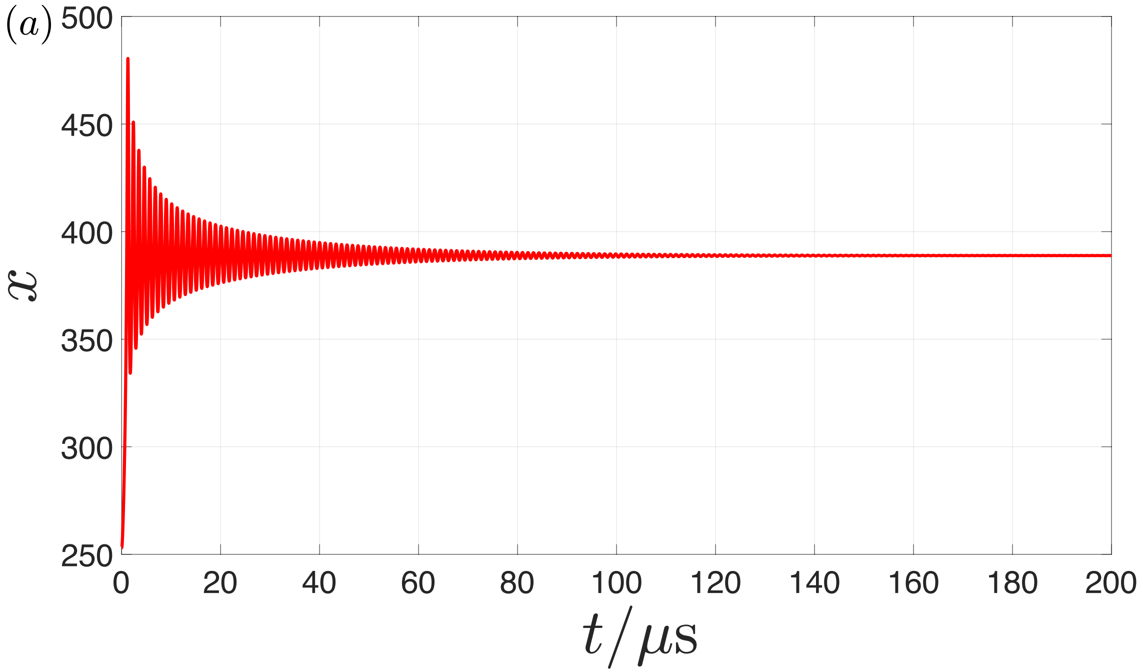
$$i_1 = G(x_1) \cdot v_1$$

$$i_2 = G(x_2) \cdot v_2$$

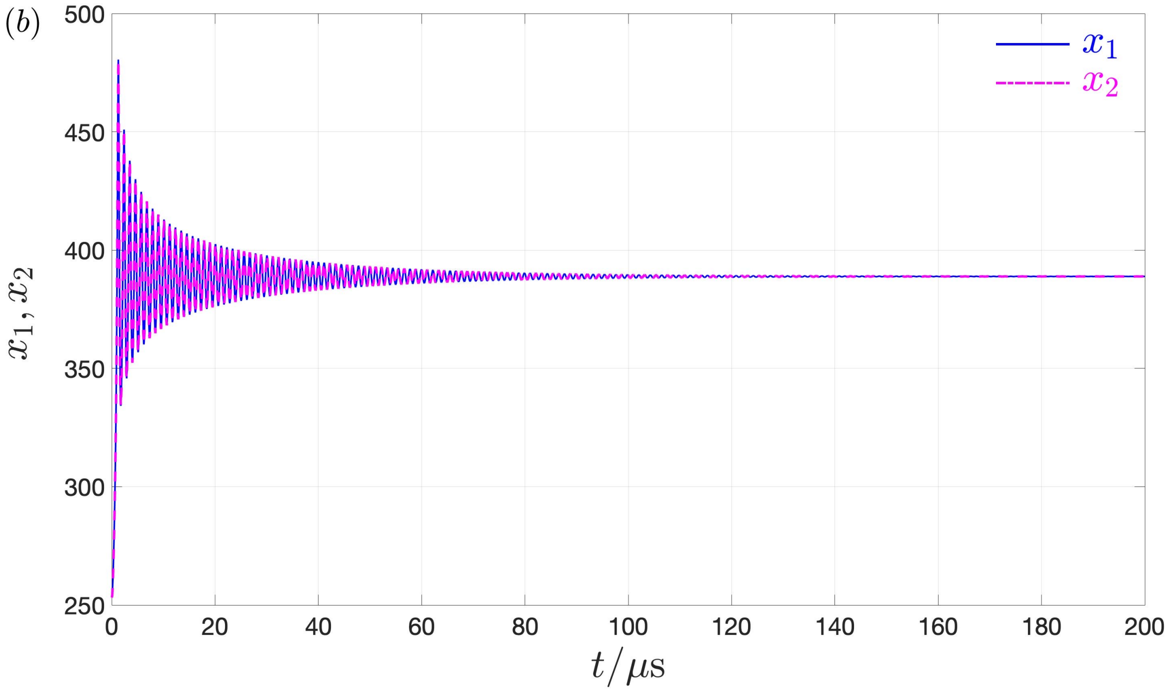
- The common expectation is that, irrespective of R_C, the memristive array would admit the homogeneous solution, where each of the two identical cells converges to the GAS operating point it would approach in the uncoupled case.
- Surprisingly, this is the case only for appropriately large R_C values.

Uncoupled cell under silence, and Homogeneous Solution of the Two-Cell Array

- Uncoupled cell, memristor bias parameters: $R_S = 100\Omega$, and $V_S = 1.3$V.
- For $\mathrm{C} = 4\mathrm{nF} < \hat{C} = 4.085\mathrm{nF}$ the uncoupled cell is poised on the EOC domain
- Coupling two identical copies of this cell via a resistor of large resistance, the resulting array displays the homogeneous solution



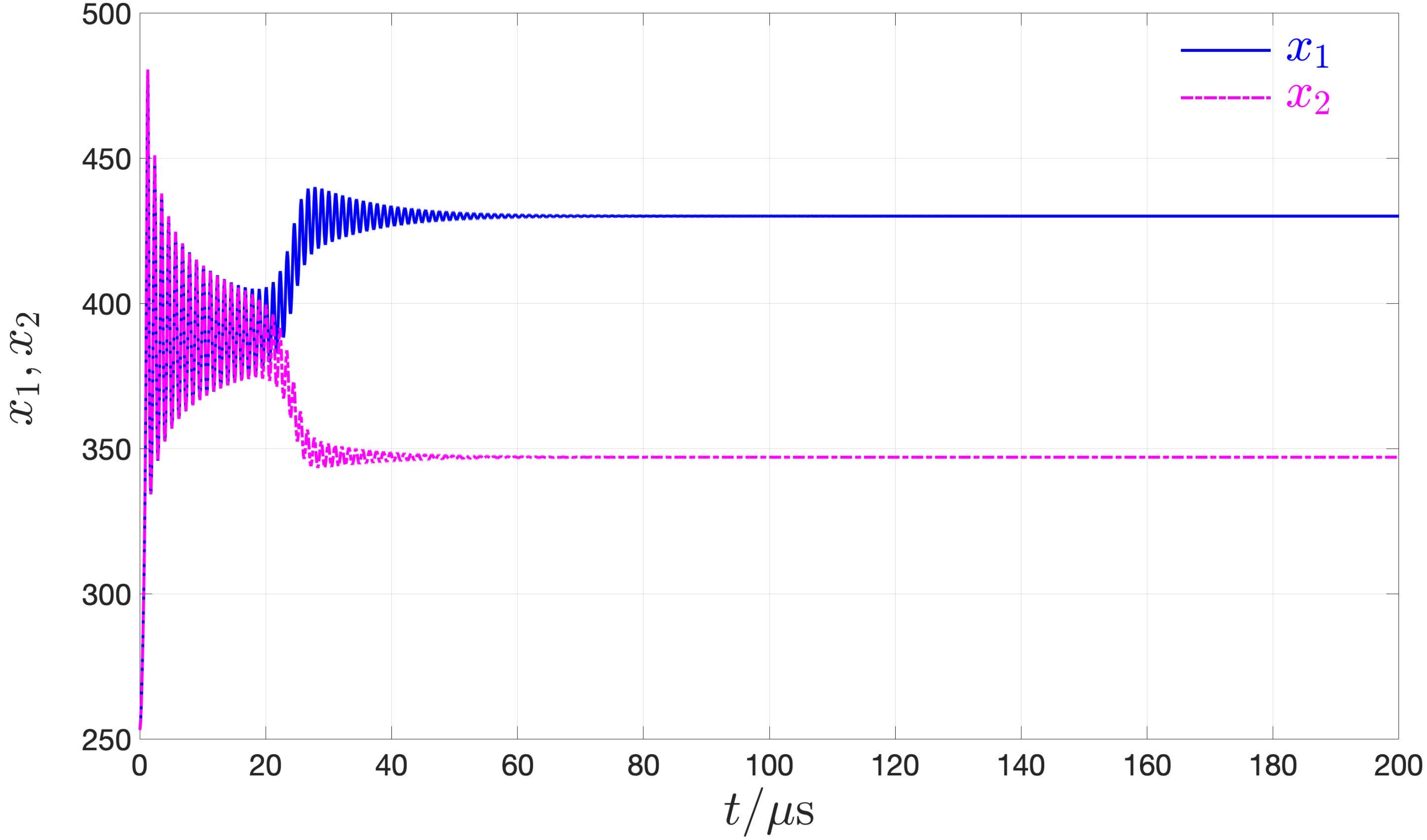
Uncoupled cell, approaching toward a globally asymptotically stable operating point (silent state) as times goes to infinity



Homogeneous solution of the two-cell array. Here $R_c = 50\Omega$

Diffusion-driven Instabilities in the Two-Cell Array: Formation of Static Turing patterns

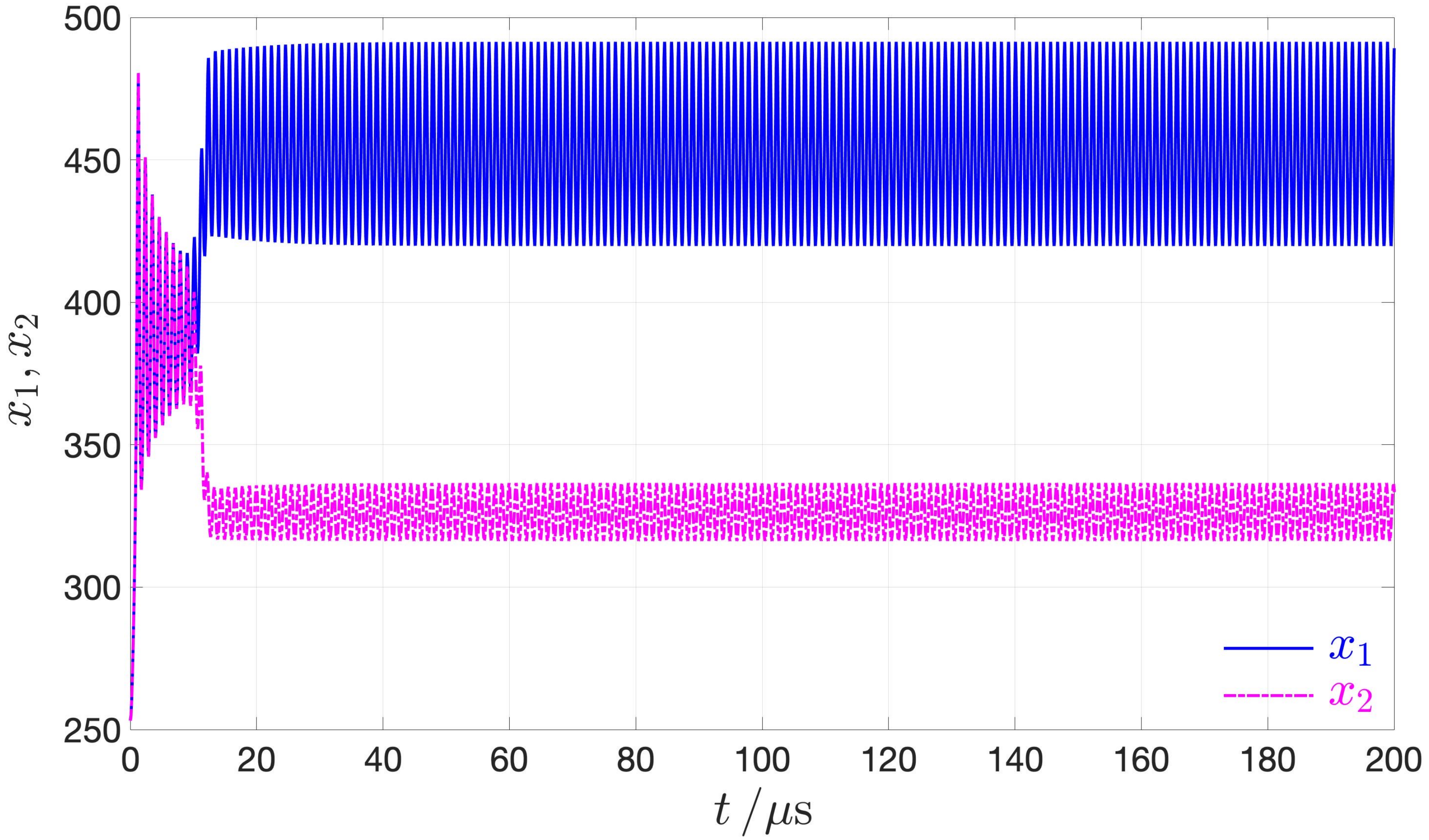
- The destabilisation of the homogeneous solution first gives way to a Turing pattern for $R_C = 49.7\Omega$



Development of an inhomogeneous static solution, i.e. a Turing pattern, in the two-cell array for $R_C = 40\Omega$

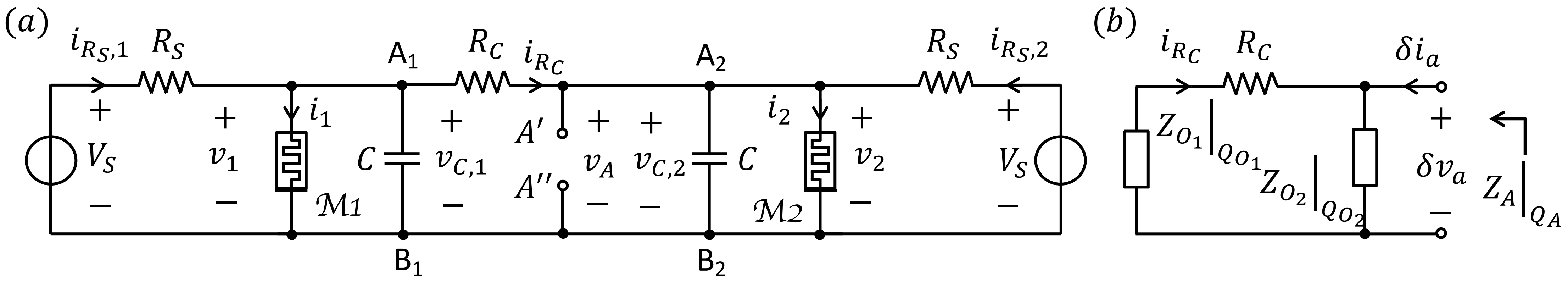
Diffusion-driven Instabilities in the Two-Cell Array: Formation of Dynamic Patterns

- Decreasing the coupling resistance further, oscillatory waveforms first develop in the cellular medium at the expenses of the inhomogeneous static solution for $R_C = 28.1\ \Omega$



Development of an oscillatory solution, i.e. a dynamic pattern, in the two-cell array for $R_C = 25\Omega$

Hints for Explaining the Origin for the Diffusion-driven Instabilities in the Two-Cell Array via Edge of Chaos Theory



- The closed-form expressioon for the small-signal impedance of the memristor array is

$$Z_a\Big|_{Q_a}(s) = \frac{\mathcal{L}\{\delta v_A(t)\}}{\mathcal{L}\{\delta i_A(t)\}} = Z_{o_2}\Big|_{Q_{o_2}}(s) \;\; || \;\; \left(R_C + Z_{o_1}\Big|_{Q_{o_1}}(s) \right)$$

where $Q_A = (Q_{O_1}, Q_{O_2})$, with $Q_{O_1} = (X_1, V_1) \neq Q_{O_2} = (X_2, V_2)$

- As R_C is decreased, a 1st bifurcation occurs, when 1 of the 4 poles of $Z_A|_{Q_A}$ for $Q_{O_1} \equiv Q_{O_2} = (X, V)$ moves to the RHP, i.e. for [3]

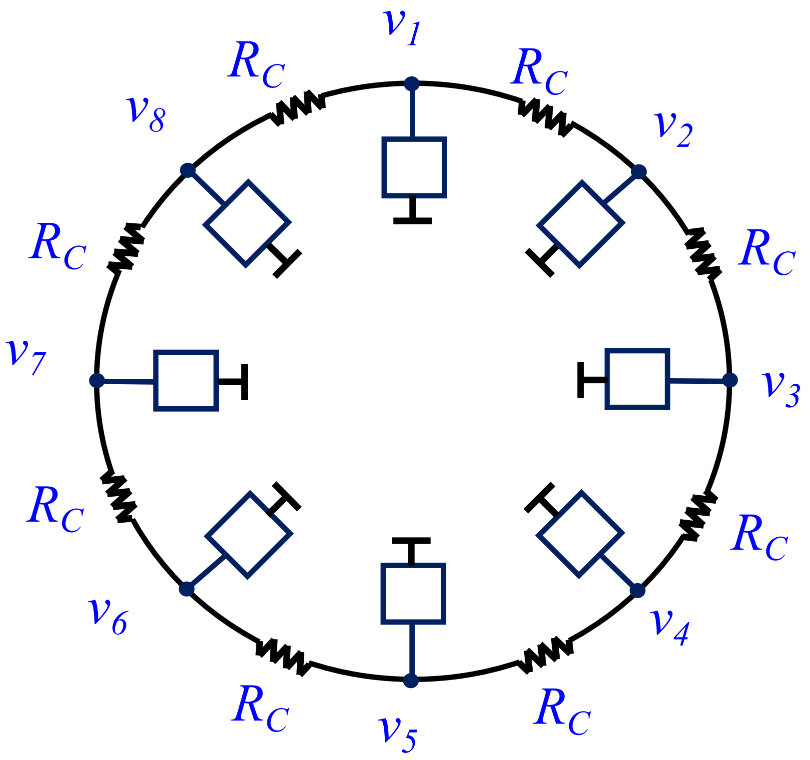
$$R_C = \frac{-2 \cdot r_2(X)}{1 + \dfrac{r_2(X)}{r_1(X) \;||\; R}}$$

Numerically, it matches the value of 49.7 Ω, computed numerically earlier, at which a Turing pattern is born in the array.

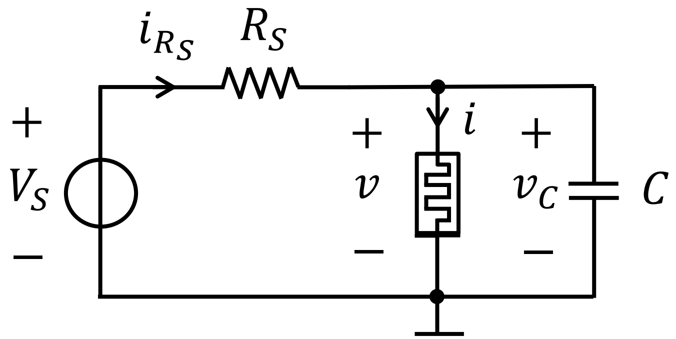
- As R_C is decreased further, a 2nd bifurcation occurs when a complex conjugate pole pair of $Z_A|_{Q_A}$ for $Q_{O_1} \neq Q_{O_2}$ move to the RHP. Using a numerical method to track the evolution of the poles of $Z_A|_{Q_A}$ on complex plane, the theory predicts the value 28.1 Ω, at which the cells were first found to pulse together, forming a dynamic pattern, in numerical simulations [3].

[3] A. Ascoli, A.S. Demirkol, R. Tetzlaff, and L.O.Chua, “Edge of Chaos behind Diffusion-Driven Instabilities in a Two-Memristor-Cell Array,” *TCAS-I*, in press

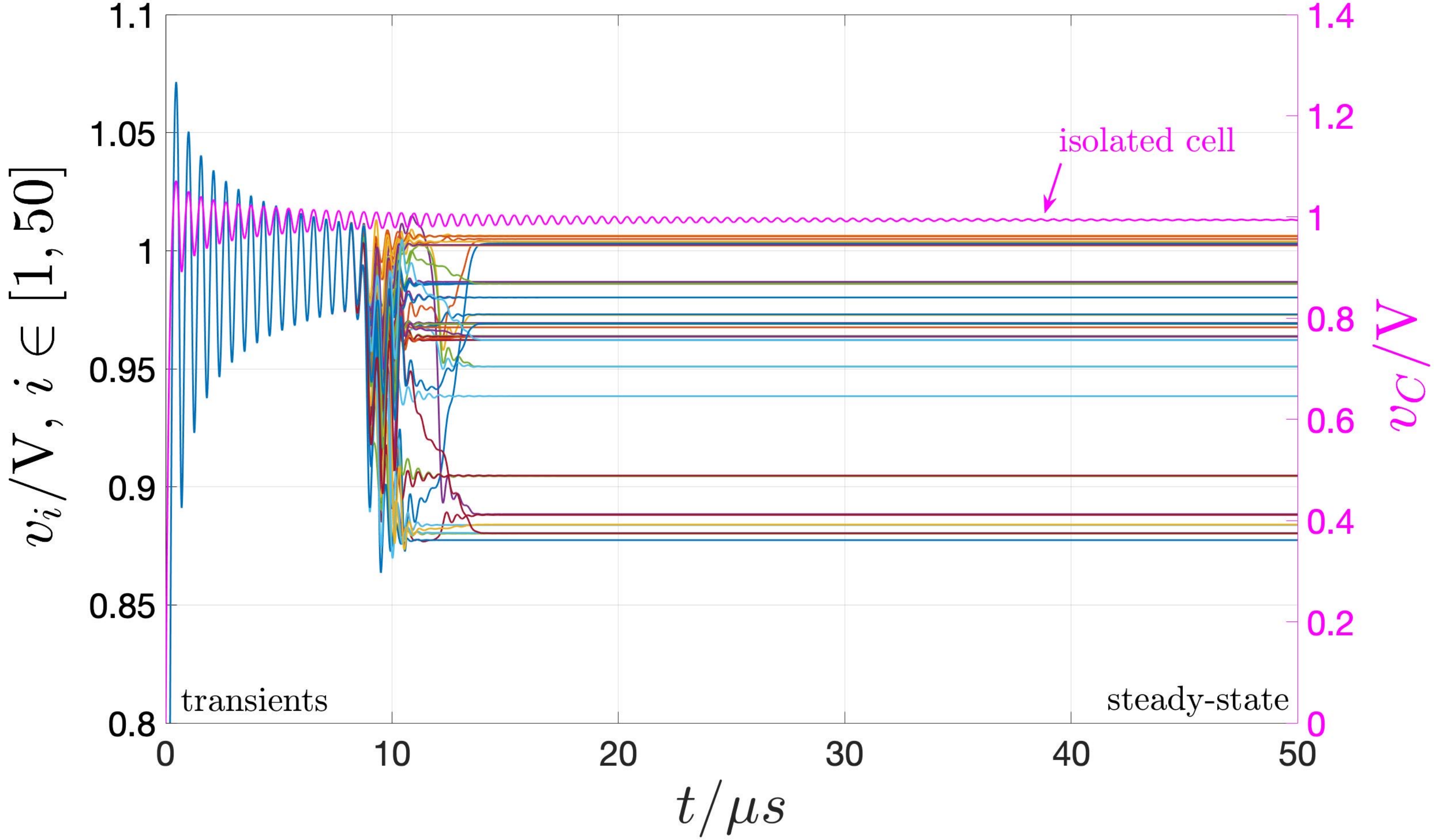
Static Turing Patterns in a Ring of Memristor Oscillators



An 8-cell ring topology



Cell circuit



Emergence of an inhomogeneous static solution (Turing pattern) in a 50-cell ring array.
In pink the uncoupled cell convergence to a locally-active and stable operating point.

Acknowledgements

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Steve Kang, University of California Santa Cruz

Leon Chua, University of California Berkeley

Chua Memristor Center

Conclusions

- I presented the simplest ever reaction-diffusion system supporting the Smale Paradox phenomenon and explained once and for all the mechanisms behind diffusion-driven static and dynamic pattern formation [1]
- Edge of Chaos may be interpreted as a new Physics Principle which extends the Second Law of Thermodynamics to Open Systems
- This Principle explains the hidden mechanisms underlying the Emergence of Heterogeneous Patterns in Homogeneous Media, what Prigogine defined as the Instability of the Homogeneous
- The theory of Local Activity [2] shall enable the development of a systematic and rigorous approach to design bio-inspired circuits with small-signal memristive amplifiers [3]-[4]
- Applications include the development of high-performance brain-like machines and biologically-plausible neuromorphic systems [5]-[6]

Thank You

[1] A. Ascoli, A.S. Demirkol, R. Tetzlaff, and L.O.Chua, “Edge of Chaos behind Diffusion-Driven Instabilities in a Two-Memristor-Cell Array,” *TCAS-I*, in press

[2] L.O. Chua, “Local activity is the origin of complexity”, Int. J. on Bifurcation and Chaos, vol. 15, no. 11, pp. 3435-3456, 2005

[3] A. Ascoli, S. Slesazeck, H. Mähne, R. Tetzlaff, and T. Mikolajick, “Nonlinear dynamics of a locally-active memristor,” *IEEE Trans. Circuits Systems–I: Reg. Papers*, vol. 62, no. 4, pp. 1165–1175, 2015

[4] A. Ascoli, S.A. Demirkol, R. Tetzlaff, S. Slesazeck, T. Mikolajick, and L.O. Chua, “On Local Activity and Edge of Chaos in a NaMLab Memristor,” Frontiers in Neuroscience, 2021

[5] A. Ascoli, R. Tetzlaff, and L.O. Chua, “Edge of Chaos: The Elan Vital of Complex Phenomena”, https://cmc-dresden.org/media/edge-of-chaos-the-elan-vital-of-complex-phenomena/

[6] A. Ascoli, A.S. Demirkol, R. Tetzlaff, and L.O.Chua, “Exploration of Edge of Chaos in Bio-Inspired Devices, Circuits, and Systems,” Proc. 17th Int. Workshop on Cellular Nanoscale Networks and their Applications (CNNA), 2021