Consiglio Nazionale delle Ricerche
Istituto di Analisi dei Sistemi ed Informatica "Antonio Ruberti"

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ROBUST OPTIMAL PLANNING OF WASTE SORTING OPERATIONS THROUGH MIXED INTEGER LINEAR PROGRAMMING

R. 21-01, March 2021

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This work has been partially supported by EU POR-FESR program of LAZIO Region on "Circular Economy and Energy" through the project REMIND – REverse Manufacturing Innovation Decision system [Grant No. B86H18000160002].

ISSN: 1128–3378

ISSN: 1128-3378 – “Collana dei rapporti dell’Istituto di Analisi dei Sistemi ed Informatica”

Research Report Series of

Istituto di Analisi dei Sistemi ed Informatica “Antonio Ruberti” – CNR

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Abstract

Waste management and circular economy objectives are worthwhile and worldwide challenges concerning both the protection of the environment and the conservation of natural resources with the aim of zero waste. A considerable attention has been directed over the last decade towards the optimization of planning procedures related to waste management in order to empower circular economy ambition. This work investigates the operations of waste recycling centers where materials are collected by a fleet of trucks and then sorted in order to be converted in secondary raw materials. The activity is characterized by low margins, difficulties to track flows and uncertainties in supplies. Indeed, waste arrivals processes are highly stochastic by nature, therefore robust optimization can also be taken in consideration for properly addressing problems of this kind of scenarios. In these settings, we propose a mixed integer linear programming model to schedule the sorting operations of each phase of the waste sorting process. The nominal problem is then enhanced by introducing robustness to supplies uncertainties making use of probabilistic bounds of constraints violations. This linear probabilistic robust approach is introduced according to Bertsimas and Sim robustness theory. The presented model can be described as a robust variant of a lot size model with non linear costs (approximated by mean of piece-wise linear functions) with the additional features of scheduling the operations and allocating the appropriate workforce dimension. The model is tested on a real world case study and results demonstrate the validity of the approach.

1. Introduction

Performances of waste management systems have been improving thanks to a noticeable commitment of decision makers and research efforts regarding the optimization of each system components. As an example, a conventional operational task addressed by research is about waste shipment and collection trucks route optimization [4][6][3][1]. In the meantime, some similar kind of optimization models have been drastically reducing transportation costs enhancing the growth of the online shopping of any sort of good. As a result, while logistic companies start serving a new magnitude of customers, also a new dimension of packaging waste started affecting the overall waste system. This leads to the need of a stronger technological and strategic decision support to packaging waste facilities in order to lower all the extra costs involved with the selective collection and sorting of this kind of waste. Not only logistic companies but also every other kind of industry generates a considerable amount of packaging waste. In Europe the Directive 2004/12/EC on packaging and packaging waste laid down the European recycling and recovery targets. In particular, official reporting on packaging waste for all EU Member States was implemented in 2007 and since then Eurostat monitors also the developments of this statistics. For example, in 2016, 170 kg of packaging waste was generated per inhabitant in the EU (varying from 55 kg per inhabitant in Croatia and 221 kg per inhabitant in Germany). Instead, from 2007 to 2016, paper and cardboard was the main packaging waste material in the EU (35.4 million tonnes in 2016) followed by plastic and glass (16.3 million tonnes for each of these waste materials in 2016). Therefore, the need of meeting the recovery and recycling targets imposed by EU law and the rising prices of raw materials used for packaging have resulted in an increasing interest in the recovery of materials from the waste streams. Moreover, the recycling industry is characterized by very low margins and high percentage of operation and logistics costs. For this reason it is critical the optimization of the process in order to turn it in an economically sustainable business. Special attention should be paid to the fact that this objective is affected by several uncertainties such as those arising in the waste streams processes. In particular, waste arrivals to sorting facilities are stochastic processes. Indeed, waste truck arrivals are subject to considerable variability that should be properly addressed when modeling scenarios including waste streams. In [5] this subject has gained interest previously. This work intends to expand the modeling power of the MILP presented in [5] by introducing robustness to data uncertainties related to waste supplies. Accordingly, the main research aim of this study is to develop a mixed integer linear programming model for planning and scheduling the packaging waste recycling operations taking into consideration also the stochastic nature of waste arrivals. This is done by introducing a protection function in each constraint according to the probabilistic robust approach presented in [2]. This approach ensures deterministic and probabilistic guarantees on constraints satisfaction and it does so in a linear framework. The model keeps supporting also other strategic decisions such as sizing the amount of processed waste and allocating the optimal number of operators for each shift of the waste sorting processes. The reminder of this document is organized as follows: Section 2 is dedicated to the problem description and the MILP formulation; Section 3 presents the experimental results; Section 4 gives some conclusions and research perspectives, while Section 5 reports acknowledgments. For a deeper description of the waste sorting process refer to Section 2.1 of [5].

2. Problem definition and modeling

In this section we describe the main operational features covered by the nominal deterministic model and present the formulation of its robust counterpart. It will clarify how the model is able to cover the principal strategic decisions of the process while properly modeling the typical production dynamics of a reverse logistic setting. The production demand of the waste facility arises from the need to program

and size the sorting operations of waste in order to balance the availability of the buffer of received material with the production and set-up costs of sorting operations and storage costs of all the inter-operational buffers. Therefore, the simultaneity of the scheduling problem and the lot sizing problem is highlighted. It is important to notice that, in the considered industrial case, costs of storage are not measurable directly. In fact, the waste stored in the buffers have neither cost nor value attributable to it that generate a variable cost of storage. The quantity stored in the buffers has no observable value since the percentages of the secondary raw materials contained in it are not known before the sorting process is complete. At the same time the level of buffer storage can be such as to constitute a criticality in terms of saturation of the storage capacity. This is particularly evident when a specific level of stock is passed. Therefore, in [5] it is considered appropriate to model this dynamic through a storage cost curve which originally included a non-linearity from the exceeding of the critical stock level. The linearity of the model is indeed guaranteed using a piece-wise linear curve that approximates the real cost curve. The indications about the threshold perceived by the waste company in relation to the customer service level can also be considered.
In the following, we introduce a mixed integer linear programming (MILP) model which defines the robust counterpart to the problem newly introduced in [5]. The basic notations that will be used in the MILP, such as parameters and indexes, are the following:

$j \in \{1,\dots,J\}$: index of the J sorting stages

$p \in \{1,\dots,P\}$: index of the P time-shifts

T : time horizon partitioned in time shifts with $t \in \{1,\dots,T\} = T_1 \cup \dots \cup T_P$

C : hourly cost of each operator

σ_t : working hours for time t determined by the corresponding shift p

$C_t = C * \sigma_t$: cost of each operator at time t

f_j : set-up cost of sorting stage j

a_t : quantity of material in kg unloaded from trucks at time t

α_j : percentage of waste processed in stage $j-1$, received in input by buffer j

S_j : maximum inventory capacity of the sorting stage buffer j

LC_j : critical stock level threshold of buffer j

ρ_j : fraction of material allowed to be left at buffer j at the end of time horizon

K_j : single operator hourly production capacity [kg/h] of sorting stage j

$SK_{j,t} = K_j * \sigma_t$: operator sorting capacity in sorting stage j, at time t

M : maximum number of operators available in each time shift

E_j : minimum number of operators to be employed in each time shift of stage j

∂h_j^i : slope of the i-th part of linearization of the buffer j stock cost curve

The nominal deterministic model consider the following variables.

$x_{j,t} \in \mathbb{Z}^+$: operators employed in the sorting stage j at time t

$u_{j,t} \in \mathbb{R}^+$: processed quantity at stage j at time t

$y_{j,t} \in \{0,1\}$: equal to 1 if stage j is activated at time t , 0 otherwise

$I_{j,t} = I^{'}_{j,t} + I^{''}_{j,t} \geq 0$: stock level of material in buffer j at time t; for each stage j the corresponding $I'_{j,t}$ and $I''_{j,t}$ represent the inventory level before and after reaching the critical threshold respectively.

$w_{j,t} \in \{0,1\}$: equal to 1 if $I^{''}_{j,t} > 0$, 0 otherwise. Indeed, this binary variables are used to model the piece-wise linear functions of the buffer stock costs.

We consider the set of parameters $a_t, t \in T$, that are subject to uncertainty taking values according to a symmetric distribution with mean equal to the nominal value a_t in the interval $[a_t - \hat{a}_t, a_t + \hat{a}_t]$. Indeed $\hat{a}_t$ is the maximum deviation of a_t. In order to meet the standard formulation of the nominal problem presented in [2], where parameters subject to uncertainties belong to inequality constraints only, the equality constraints of [5] regarding waste arrivals a_t are reformulated to turn them into inequality constraints. This is performed considering for each period t the sum of all the received and processed quantities of waste up to that period, as in constraints (5),(6),(7),(8) of the formulation presented in this section. According to the robust approach presented in [2], a parameter Γ_i is introduced for each constraint i holding one or more uncertainty coefficients. Γ_i is not necessarily integer and takes values in the interval $[0, |J_i|]$ where J_i is the set of the coefficients of constraint i being subject to uncertainty. The nominal problem presented in [5] presents only one set of T constraints considering the coefficients a_t and these are the ones reformulated as inequality constraints. Therefore we get $\Gamma \in R^T_+$, and because of this reformulation $|J_t| = t \ \forall t \in \{1, \dots, T\}$. For each period t, Γ_t represents the number of coefficients that we consider as allowed to vary within their interval, ergo we consider nature behaving like only a subset of the coefficients will change with respect to their nominal value. Indeed, as affirmed in [2], it is unlikely that all $|J_t|$ will change; so the idea of conservative robustness is to be protected against all cases that up to $\lfloor \Gamma_t \rfloor$ of these coefficients are allowed to change, and one coefficient a_t changes by $(\Gamma_t - \lfloor \Gamma_t \rfloor)\hat{a}_t$. Note that when $\Gamma_t = 0 \ \forall t \in \{1, \dots, T\}$ we get the nominal deterministic scenario, while setting $\Gamma_t = |J_t| = t \ \forall t \in \{1, \dots, T\}$ represents solving the problem of the worst case scenario. It is clear then that by varying Γ the level of robustness can be flexibly adjusted against the level of conservatism of the solution. Considering the peculiar structure of the constraints including a_t is important: because of the telescopic expansion of each set J_t as t goes from 1 to T (i.e. $|J_{t+1}| = |J_t| + 1$), we consistently constraint Γ_t to be bigger or equal to Γ_{t-1}.
In the following, we present all the additional variables and parameters that are necessary to introduce the robustness protection functions presented in [2] and formulate the robust counterpart of the model presented in [5]:

$\varepsilon_t \in \mathbb{R}^+$: extra variables multiplying $a_t \ \forall t \in T$. These variables are introduced in order to have a variable multiplying the only set of parameters that are affected by uncertainty. These are indeed constrained to be equal to 1 $\forall t \in T$.

$z_t \in \mathbb{R}^+$: variable resulted of duality within Bertsimas and Sim robustness theory; when multiplied by Γ_t provides its overall contribution to the protection function of constraint t.

$p_{t,k} \in \mathbb{R}^+$: variable resulted of duality within Bertsimas and Sim robustness theory; provides its contribution to the protection function of constraint t with respect to the specific coefficient a_k.

$s_t \in \mathbb{R}^+$: variable resulted of duality and Bertsimas and Sim robustness theory; multiplied by $\hat{a}_t$ sets the lower bound of the protection function contribution in each constraint t.

Γ_t : parameter to adjust the level of robustness of each period t.

Considering a case study where $J = 2$ sorting stages, for the 1st sorting phase, $u_{1,t} \geq 0$ and $x_{1,t} \in \{0,1\}$ represent the quantity of material to be selected and decision to activate the process respectively at time t. For the 2nd sorting phase, $u_{2,t} \geq 0$ and $x_{2,t} \in \{0,1\}$ represent the quantity of material to be selected and decision to activate the process respectively at time t. $I_{1,t}, I'_{1,t}, I''_{1,t} \geq 0$ are the inventory levels at 1st phase sorting buffer while $I_{2,t}, I'_{2,t}, I''_{2,t} \geq 0$ are inventory levels at 2nd phase sorting buffer. As previously stated, w_1 and w_2 are used to model the piece-wise linear functions of the buffer stock costs. In detail $w_1 = 0$ if $I'_{1,t} < LC$, 1 if $I'_{1,t} = LC$ and $I''_{1,t} > 0$; similarly $w_2 = 0$ if $I'_{2,t} < LC$, 1 if $I'_{2,t} = LC$ and $I''_{2,t} > 0$. The model minimizes the sum of sorting and holding costs and is detailed as following:

$$\min Z = \sum_{j\in J}\sum_{t\in T} C_t x_{j,t} + \sum_{j\in J}\sum_{t\in T} f_j y_{j,t} + \sum_{j\in J}\sum_{t\in T}\left(\partial h_j^1 I^{'}_{j,t} + \partial h_j^2 I^{''}_{j,t}\right) \tag{1}$$

s.t.

$$E_j\, y_{j,t} \leq x_{j,t} \leq M\, y_{j,t} \qquad \forall j \in J, t \in T_p, p \in P \tag{2}$$

$$\sum_{j\in J} x_{j,t} \leq M \qquad \forall t \in T \tag{3}$$

$$u_{j,t} \leq SK_{j,t}\, x_{j,t} \qquad \forall j \in J, t \in T \tag{4}$$

$$I_{1,0} + \sum_{k=1}^{t} a_k \varepsilon_k - \sum_{k=1}^{t} u_{1,k} + z_t \Gamma_t + \sum_{k=1}^{t} p_{t,k} \leq S_1 \qquad \forall t \in T \tag{5}$$

$$I_{1,0} + \sum_{k=1}^{t} a_k \varepsilon_k - \sum_{k=1}^{t} u_{1,k} \geq 0 \qquad \forall t \in T \tag{6}$$

$$I_{1,0} + \sum_{k=1}^{T} a_k \varepsilon_k - \sum_{k=1}^{T} u_{1,k} + z_T \Gamma_T + \sum_{k=1}^{T} p_{T,k} \leq \rho_1\, LC_1 \tag{7}$$

$$I_{1,t} = I_{1,0} + \sum_{k=1}^{t} a_k \varepsilon_k - \sum_{k=1}^{t} u_{1,k} + z_t \Gamma_t + \sum_{k=1}^{t} p_{t,k} \qquad \forall t \in T \tag{8}$$

$$I_{j,t} = I_{j,t-1} - u_{j,t} + \alpha_j\, u_{j-1,t} \qquad \forall t \in T, j \in J \setminus 1 \tag{9}$$

$$I_{j,t} = I^{'}_{j,t} + I^{''}_{j,t} \qquad \forall j \in J, t \in T \tag{10}$$

$$LC_j\, w_{j,t} \leq I^{'}_{j,t} \leq LC_j \qquad \forall j \in J, t \in T \tag{11}$$

$$0 \leq I^{''}_{j,t} \leq (S_j - LC_j)\, w_{j,t} \qquad \forall j \in J, t \in T \tag{12}$$

$$I_{j,T} \leq \rho_j\, LC_j \qquad \forall j \in J \setminus 1 \tag{13}$$

$$z_t + p_{t,k} \geq \widehat{a}_t\, s_t \qquad \forall t \in T, k \in \{0,...,t\} \tag{14}$$

$$-s_t \leq \varepsilon_t \leq s_t \qquad \forall t \in T \tag{15}$$

$$\varepsilon_t = 1 \qquad \forall t \in T \tag{16}$$

$$x_{j,t} \in \mathbb{Z}^+ \qquad \forall j \in J, t \in T \tag{17}$$

$$u_{j,t} \in \mathbb{R}^+ \qquad \forall j \in J, t \in T \tag{18}$$

$$y_{j,t} \in \{0,1\} \qquad \forall j \in J, t \in T \tag{19}$$

The objective function (1) defines the minimization of the sum of the three cost terms, which are sorting, setup, and inventory costs respectively. (2) and (3) bounds the number of workers that can be assigned to each sorting station and to each time shift. Constraints (4) limit the quantity sorted $u_{j,t}$ to the sorting capacity dependent on the number of workers $x_{j,t}$. The following constraint sets of (5)(6)(7)(8)(9) define and limit the inventories: constraint (5) defines the inventory for the first buffer, considering the cumulative inbound material a_t up to period t, the overall sorted material u_{1t} up to period t, and the uncertainties protection function made of the joint contribution of $z_t\Gamma_t$ and the sum of $p_{t,k}$ for $k \in \{1,\ldots,t\}$. Constraint (6) sets the lower bound of the inventory for each period and (7) imposes the maximum unsorted material allowed to be left at the end of the planning period for the first buffer, as constraint (13) does for all other subsequent buffers. Equality constraint (8) allows the inventory of the main buffer (i.e. buffer no. 1) to be considered in the corresponding piece-wise linear part of the cost function. Constraint (9) defines the inventory for the other buffers corresponding to $j > 1$. Indeed (9) outlines the waste flow across the sorting stages that follow one another: each subsequent inter-operational buffer j receives by the previous sorting stage $j-1$ a quantity of waste equal to a α_j percentage of the waste processed in stage $j-1$. Constraint sets (10), (11), and (12) define the piece-wise linear functions for inventories; in these constraints, level S_j and maximum capacity LC_j are connected with the inventory levels through the variable $w_{j,t}$. Constraints (14) and (15) resulted from duality in [2] robustness theory; where (14) sets the lower bound of the protection function contribution in constraints (5) and (7).

3. Experimental results

This section holds the main results from the studied scenarios described in the following. All instances are created by a real-world case study from a waste sorting plant located next to Rome, Italy. We solved each scenario by applying the Gurobi 9.0 solver to the MILP model. This has been performed in order to test the model response to different levels of robustness (i.e. different Γ selection), the corresponding price of robustness (i.e. the optimality reduction w.r.t the deterministic scenario) with respect to different weeks of scheduling time horizons. Figure 1 provides a first look at the model reaction to three different scenarios: deterministic case, an intermediate level of protection and the worst case scenario. Is evident that in the deterministic case the protection function value remain null for each period, almost like the first buffer stock level. Indeed the production marginal cost is less than the storage marginal cost resulting in the processing of waste as soon as arrives at the sorting facility. This is the reason why considering constraint (8) the protection function value equals for each period the first buffer stock level, and this applies for each protection scenario. Therefore we can attribute a cost to the protection function as the extra cost related to an higher level of stock in first buffer receiving the uncertain amount of waste. The robustness performance and the corresponding additional cost definitely depend on the protection strategy of choosing the vector $\Gamma \in R^T_+$ s.t. $\Gamma_t \geq \Gamma_{t-1} \forall t \in T$. In Figure 1 the intermediate protection relies on a moderate still continuous increase of Γ_t. This approach represents a cumulative sum of protection over the risk considered across the time horizon. Dealing with robustness to reverse demand uncertainties in a scheduling problem setting is a suggestion to consider the seasonality of the stochastic behaviour of the coefficients when dealing with the strategy of choosing $\Gamma \in R^T_+$. In the considered real case application the parameters a have a one week period (i.e. $t_{period} = 12$ when $P = 2$ working shift a day for six working days). Therefore a good approach is increasing Γ_t for $t \in \{1,..,t_{period}\}$ and keeping the maximum Γ_{period} for the rest of the time horizon. Figure 2 shows an example with a three weeks time horizon (i.e. T = 36).
The price of robustness (the optimality reduction w.r.t. the nominal deterministic problem) is tested over twenty protection magnitudes with respect to different time horizons from one to four weeks. All Γ selections linearly increase with different slopes from minimum to maximum risk protections as shown

8.

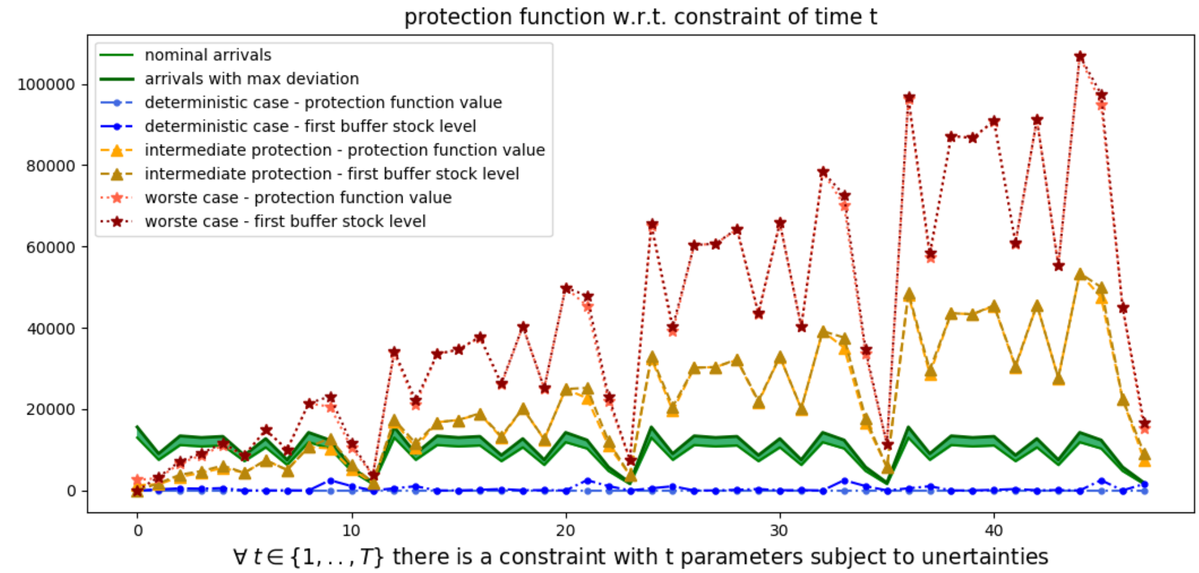

Figure 1: Illustration of protection function and stock level evolution over three different uncertainties protection scenarios

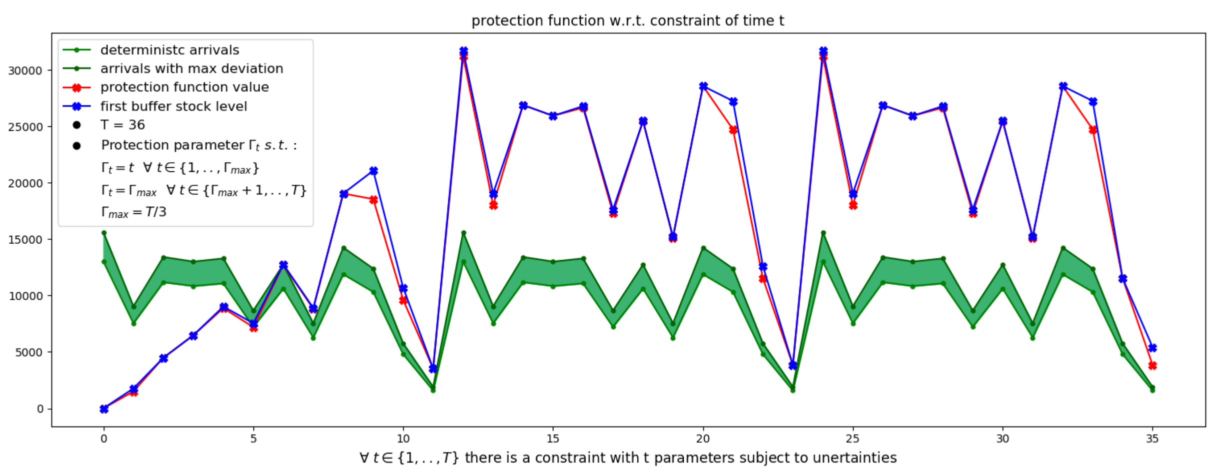

Figure 2: Using demand period for Γ selection strategy

in Figure 3.
Results concerning the price of robustness are presented in Figure 4. It is clear that the evolution of the price paid for risk protection remains reasonable and its evolution with respect to the protection scenarios strictly depend on the strategical selection of Γ. Indeed a linear evolution of the price is obtained with a linear expansion of Γ components.

4. Conclusions

We advanced a tuned version of the model presented in [5] with additional complexity due to the introduction of robustness on the most critical parameters values. The formulation keeps supporting all the original strategic decisions that are critical in the business considered. This robust counterpart showed

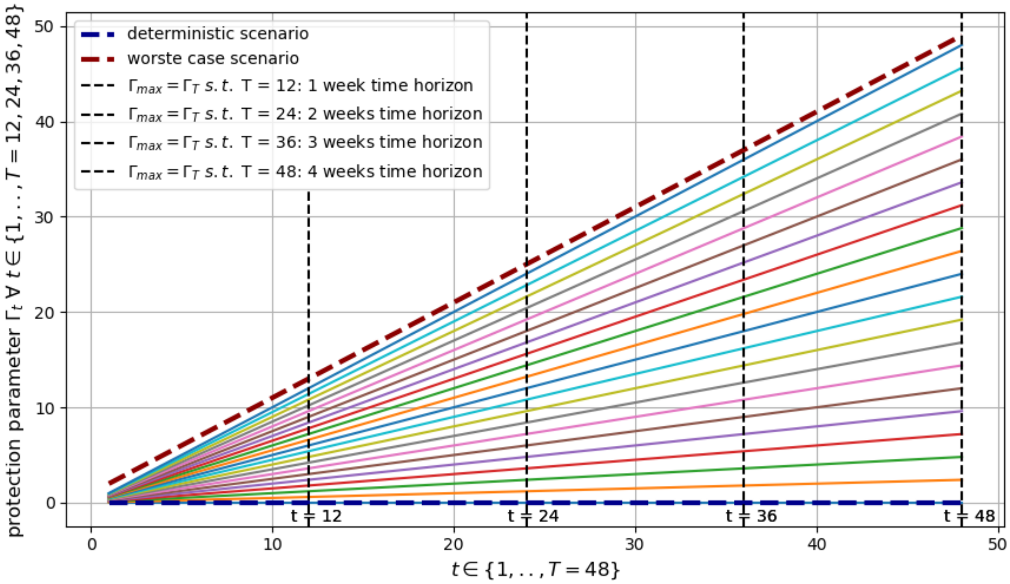

Figure 3: protection magnitude scenarios: from deterministic to worste case

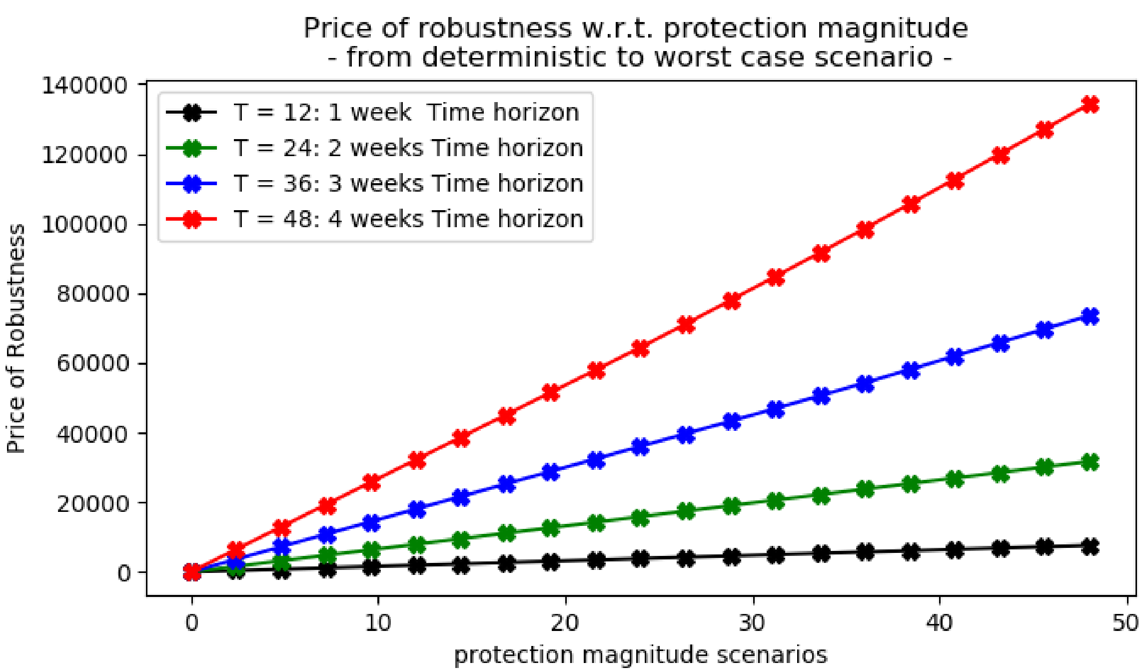

Figure 4: Price of robustness results

a good adjustable protection capacity when used in a real-world application. Results concerning the controllable price of robustness in the considered case study are also encouraging. Indeed, for the company level of service, this economical and controllable improvement is highly remarkable, taking into account the low margin of the activity. Future works may consider to introduce more complexity in the formulation, such as considering production capacity dependent on the size of working teams.

5. Acknowledgment

This work has been partially supported by EU POR-FESR program of LAZIO Region on ”Circular Economy and Energy” through the project REMIND – REverse Manufacturing Innovation Decision system [Grant No. B86H18000160002].

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