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ON 51 FORBIDDEN CONFIGURATIONS FOR SELF-COMPLEMENTARY CIRCULAR CODES

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Abstract

If a code belongs to the set of the maximal self-complementary circular codes then it verifies or that both its two permuted codes are circular codes or that they both admit a necklace, so that no one of them is a circular code.

1. Introduction

Circular self-complementary codes, found in [13], and studied in [10], are 528. To every self-complementary circular code X_0 we can naturally associate two sets X_1 and X_2 (see the following paragraph for the formal definitions). We know that (see [10]) for exactly 216 of the 528 self-complementary circular codes both X_1 and X_2 are circular codes. For the remaining 312 codes, the circularity of X_1 and of X_2 is not investigated in [10] and, prior, three cases can occur:
a) X_1 is a circular code and X_2 is not a circular code;
b) X_2 is not a circular code and X_2 is a circular code;
c) X_1 and X_2 are not circular codes.
The purpose of this work is to prove that only case c) is verified. This result has been obtained with a detailed analysis of 51 "forbidden configurations" corrisponding to 51 propositions that are enunciated and proved in the following.
Considering together what is proved in [10] and what is proved in this work, we have that the 528 self-complementary circular codes are divided in two classes of 216 and 312 circular codes respectively, the first contains circular codes X_0 with X_1 and X_2 both circular codes while the second contains codes X_0 with X_1 and X_2 both non circular codes. Therefore the following proposition holds.
Proposition. If X_0 is a self-complementary circular code, then or X_1 and X_2 are both circular codes or X_1 and X_2 admit both a necklace (and, cosequently, they are not circular codes).

2. Definitions

For the classical notions of *alphabet, empty word, length, factor, proper factor, prefix, proper prefix, suffix, proper suffix, alphabetical order*, we refer to the reader to [3]. Let $\mathcal{A}_4 = \{A, C, G, T\}$ denote the genetic alphabet, alphabetically ordered with
$A < C < G < T$, $\mathcal{A}_4^*$ (respectively $\mathcal{A}_4^+$) the set of words (respectively non empty words) over $\mathcal{A}_4$ and $\mathcal{A}_4^3$ the set of the 64 words of length three (or *trinucleotides*).
We now recall two important genetic maps, the notions of code and circular code, and the property of being C^3 for a code.

Definition 1. *The complementarity map* $\mathcal{C}$: $\mathcal{A}_4^+ \to \mathcal{A}_4^+$ *is defined by* $\mathcal{C}(A) = T$, $\mathcal{C}(T) = A$, $\mathcal{C}(C) = G$ *and* $\mathcal{C}(G) = C$ *and by* $\mathcal{C}(uv) = \mathcal{C}(v)\mathcal{C}(u)$ *for all* $u, v \in \mathcal{A}_4^+$, *e.g.,* $\mathcal{C}(AAC) = GTT$. *This map on words is naturally extended to word sets: a complementary trinucleotide set is obtained by applying the complementarity map* $\mathcal{C}$ *to all its trinucleotides.*

Definition 2. *The circular permutation map* $\mathcal{P}$: $\mathcal{A}_4^3 \to \mathcal{A}_4^3$ *permutes circularly each trinucleotide* $l_1l_2l_3$ *as follows* $\mathcal{P}(l_1l_2l_3) = l_2l_3l_1$. *The k-th iterate of* $\mathcal{P}$ *is denoted* $\mathcal{P}^k$. *This map on words is also naturally extended to word sets: a permuted trinucleotide set is obtained by applying the circular permutation map* $\mathcal{P}$ *to all its trinucleotides.*

Definition 3. *Code: A set* X_0 *of words is a code if, for each* $x_1, \ldots, x_n, x'_1, \ldots, x'_m \in X$, $n, m \geq 1$, *the condition* $x_1 \cdots x_n = x'_1 \cdots x'_m$ *implies* $n = m$ *and* $x_i = x'_i$ *for* $i = 1, \ldots, n$.

Definition 4. *A trinucleotide code* X_0 *is circular if, for each* $x_1, \ldots, x_n, x'_1, \ldots, x'_m \in X$, $n, m \geq 1$, $p \in \mathcal{A}_4^*$, $s \in \mathcal{A}_4^+$, *the conditions* $sx_2 \cdots x_np = x'_1 \cdots x'_m$ *and* $x_1 = ps$ *imply* $n = m$, $p = \varepsilon$ *(empty word) and* $x_i = x'_i$ *for* $i = 1, \ldots, n$.

Definition 5. *A trinucleotide code X_0 is self-complementary if, for each $y \in X_0$, $\mathcal{C}(y) \in X_0$.*

Definition 6. *A trinucleotide code X_0 is C^3 self-complementary if X_0, $X_1 = \mathcal{P}(X_0)$ and $X_2 = \mathcal{P}^2(X_0)$ are circular codes satisfying the following properties: $X_0 = \mathcal{C}(X_0)$ (self-complementary), $\mathcal{C}(X_1) = X_2$ and $\mathcal{C}(X_2) = X_1$.*

The concept of a necklace was introduced by Pirillo for circular codes in [12] to have an equivalent condition for a trinucleotide code of being circular.

Let $l_1, l_2, \ldots, l_{n-1}, l_n, \ldots$ be letters in $\mathcal{A}_4$, $d_1, d_2, \ldots, d_{n-1}, d_n, \ldots$ diletters in $\mathcal{A}_4^2$ and $n \geq 2$ an integer.

Definition 7. *Letter Diletter Continued Necklaces (LDCN): We say that the ordered sequence $l_1, d_1, l_2, d_2, \ldots, d_{n-1}, l_n, d_n, l_{n+1}$ is an $(n+1)LDCN$ for a subset $X \subset \mathcal{A}_4^3$ if $l_1d_1, l_2d_2, \ldots, l_nd_n \in X$ and $d_1l_2, d_2l_3, \ldots, d_{n-1}l_n, d_nl_{n+1} \in X$.*

$\mathbf{l_1}$	$\mathbf{d_1}$	$\mathbf{l_2}$	$\mathbf{d_2}$	$\mathbf{l_3}$	$\cdots$	$\mathbf{d_{n-1}}$	$\mathbf{l_{n-1}}$	$\mathbf{d_n}$	$\mathbf{l_{n+1}}$

Proposition 2.1. *[12]. Let X be a trinucleotide code. The following conditions are equivalent.*
(i) X is circular code.
(ii) X has no $5LDCN$.

$\mathbf{l_1}$	$\mathbf{d_1}$	$\mathbf{l_2}$	$\mathbf{d_2}$	$\mathbf{l_3}$	$\mathbf{d_3}$	$\mathbf{l_4}$	$\mathbf{d_4}$	$\mathbf{l_5}$

Now we use this proposition to prove that for every of the 312 non C^3, maximal, self-complementary circular codes, both X_1 and X_2 admit a $5LDCN$, so that nor X_1 neither X_2 are circular codes.

$a = \{AAC, GTT\}$	$b = \{AAG, CTT\}$	$c = \{AAT, ATT\}$	$d = \{ACA, TGT\}$
$e = \{ACC, GGT\}$	$f = \{ACG, CGT\}$	$g = \{ACT, AGT\}$	$h = \{AGA, TCT\}$
$i = \{AGC, GCT\}$	$j = \{AGG, CCT\}$	$k = \{ATC, GAT\}$	$l = \{ATG, CAT\}$
$m = \{CAA, TTG\}$	$n = \{CAC, GTG\}$	$o = \{CAG, CTG\}$	$p = \{CCA, TGG\}$
$q = \{CCG, CGG\}$	$r = \{CGA, TCG\}$	$s = \{CTA, TAG\}$	$t = \{CTC, GAG\}$
$u = \{GAA, TTC\}$	$v = \{GAC, GTC\}$	$w = \{GCA, TGC\}$	$x = \{GCC, GGC\}$
$y = \{GGA, TCC\}$	$z = \{GTA, TAC\}$	$z' = \{TAA, TTA\}$	$z'' = \{TCA, TGA\}$

Table 1. Partition of $\mathcal{A}_4^3 \setminus \{AAA, CCC, GGG, TTT, ATA, TAT, CGC, GCG\}$ into 28 self-complementary pairs.

Proposition 2.2. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the couple $\alpha_1 = \{a,p\} = \{AAC, GTT, CCA, TGG\}$, then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. If $\alpha_1 = \{a,p\} = \{AAC, GTT, CCA, TGG\} \subset X_0$ then

$$\mathcal{P}(\{AAC, GTT, CCA, TGG\}) = \{ACA, TTG, CAC, GGT\} \subset X_1,$$

and we have that

A	**CA**	**C**	**AC**	**A**	**CA**	**C**	**AC**	**A**

is a 5LDCN in X_1,
or, shifting,

C	**AC**	**A**	**CA**	**C**	**AC**	**A**	**CA**	**C**

is a 5LDCN in X_1, so that X_1 is not a circular code;
we have also that

$$\mathcal{P}^2(\{AAC, GTT, CCA, TGG\}) = \{CAA, TGT, ACC, GTG\} \subset X_2,$$

so that the dual of the first necklace above (that is the necklace to which the complementarity map $\mathcal{C}$ is applied)

T	**GT**	**G**	**TG**	**T**	**GT**	**G**	**TG**	**T**

is a 5LDCN in X_2,
or, similarly, shifting,

G	**TG**	**T**	**GT**	**G**	**TG**	**T**	**GT**	**G**

is a 5LDCN in X_2, and we conclude that also X_2 is not a circular code.

Proposition 2.3. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the couple $\alpha_2 = \{b,y\} = \{AAG, CTT, GGA, TCC\}$, then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.2): here, similarly, we have that

$$A, GA, G, AG, A, GA, G, AG, A$$

is a 5LDCN in X_1, and its dual necklace

$$T, CT, C, TC, T, CT, C, TC, T$$

is a 5LDCN in X_2.

Proposition 2.4. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the couple $\alpha_3 = \{e,m\} = \{ACC, GGT, CAA, TTG\}$, then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.2): here, similarly, we have that

$$\boxed{\mathbf{G}}\boxed{\mathbf{TG}}\boxed{\mathbf{T}}\boxed{\mathbf{GT}}\boxed{\mathbf{G}}\boxed{\mathbf{TG}}\boxed{\mathbf{T}}\boxed{\mathbf{GT}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{C}}\boxed{\mathbf{AC}}\boxed{\mathbf{A}}\boxed{\mathbf{CA}}\boxed{\mathbf{C}}\boxed{\mathbf{AC}}\boxed{\mathbf{A}}\boxed{\mathbf{CA}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_2.

Proposition 2.5. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the couple $\alpha_4 = \{j,u\} = \{AGG, CCT, GAA, TTC\}$, then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.2): here, similarly, we have that

$$\boxed{\mathbf{C}}\boxed{\mathbf{TC}}\boxed{\mathbf{T}}\boxed{\mathbf{CT}}\boxed{\mathbf{C}}\boxed{\mathbf{TC}}\boxed{\mathbf{T}}\boxed{\mathbf{CT}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{G}}\boxed{\mathbf{AG}}\boxed{\mathbf{A}}\boxed{\mathbf{GA}}\boxed{\mathbf{G}}\boxed{\mathbf{AG}}\boxed{\mathbf{A}}\boxed{\mathbf{GA}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_2.

Proposition 2.6. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_1 = \{a,r,w\} = \{AAC, GTT, CGA, TCG, GCA, TGC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. If $\beta_1 = \{a,r,w\} = \{AAC, GTT, CGA, TCG, GCA, TGC\} \subset X_0$ then

$$\mathcal{P}(\{AAC, GTT, CGA, TCG, GCA, TGC\}) = \{ACA, TTG, GAC, CGT, CAG, GCT\} \subset X_1,$$

and we have that

$$\boxed{\mathbf{A}}\boxed{\mathbf{CA}}\boxed{\mathbf{G}}\boxed{\mathbf{AC}}\boxed{\mathbf{A}}\boxed{\mathbf{CA}}\boxed{\mathbf{G}}\boxed{\mathbf{AC}}\boxed{\mathbf{A}}$$

is a 5LDCN in X_1,
or, shifting,

$$\boxed{\mathbf{G}}\boxed{\mathbf{AC}}\boxed{\mathbf{A}}\boxed{\mathbf{CA}}\boxed{\mathbf{G}}\boxed{\mathbf{AC}}\boxed{\mathbf{A}}\boxed{\mathbf{CA}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_1, so that X_1 is not a circular code;
we have also that

$$\mathcal{P}^2(\{AAC, GTT, CGA, TCG, GCA, TGC\}) = \{CAA, TGT, ACG, GTC, AGC, CTG\} \subset X_2,$$

so that the dual of the first necklace above (that is the necklace to which the complementarity map $\mathcal{C}$ is applied)

T | GT | C | TG | T | GT | C | TG | T

is a 5LDCN in X_2,
or, similarly, shifting,

C | TG | T | GT | C | TG | T | GT | C

is a 5LDCN in X_2, and we conclude that also X_2 is not a circular code.

Proposition 2.7. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_2 = \{a,s,z"\} = \{AAC, GTT, CTA, TAG, TCA, TGA\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

A | CA | T | AC | A | CA | T | AC | A

is a 5LDCN in X_1, and its dual necklace

T | GT | A | TG | T | GT | A | TG | T

is a 5LDCN in X_2.

Proposition 2.8. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_3 = \{b,r,w\} = \{AAG, CTT, CGA, TCG, GCA, TGC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

A | GA | C | AG | A | GA | C | AG | A

is a 5LDCN in X_1, and its dual necklace

T | CT | G | TC | T | CT | G | TC | T

is a 5LDCN in X_2.

Proposition 2.9. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_4 = \{b,z,z"\} = \{AAG, CTT, GTA, TAC, TCA, TGA\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

A | GA | T | AG | A | GA | T | AG | A

is a 5LDCN in X_1, and its dual necklace

T | CT | A | TC | T | CT | A | TC | T

is a 5LDCN in X_2.

Proposition 2.10. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_5 = \{c,s,z''\} = \{AAT, ATT, CTA, TAG, TCA, TGA\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

$$\boxed{\mathbf{A}}\boxed{\mathbf{TA}}\boxed{\mathbf{C}}\boxed{\mathbf{AT}}\boxed{\mathbf{A}}\boxed{\mathbf{TA}}\boxed{\mathbf{C}}\boxed{\mathbf{AT}}\boxed{\mathbf{A}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{T}}\boxed{\mathbf{AT}}\boxed{\mathbf{G}}\boxed{\mathbf{TA}}\boxed{\mathbf{T}}\boxed{\mathbf{AT}}\boxed{\mathbf{G}}\boxed{\mathbf{TA}}\boxed{\mathbf{T}}$$

is a 5LDCN in X_2.

Proposition 2.11. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_6 = \{c,z,z''\} = \{AAT, ATT, GTA, TAC, TCA, TGA\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

$$\boxed{\mathbf{A}}\boxed{\mathbf{TA}}\boxed{\mathbf{G}}\boxed{\mathbf{AT}}\boxed{\mathbf{A}}\boxed{\mathbf{TA}}\boxed{\mathbf{G}}\boxed{\mathbf{AT}}\boxed{\mathbf{A}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{T}}\boxed{\mathbf{AT}}\boxed{\mathbf{C}}\boxed{\mathbf{TA}}\boxed{\mathbf{T}}\boxed{\mathbf{AT}}\boxed{\mathbf{C}}\boxed{\mathbf{TA}}\boxed{\mathbf{T}}$$

is a 5LDCN in X_2.

Proposition 2.12. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_7 = \{e,l,s\} = \{ACC, GGT, ATG, CAT, CTA, TAG\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

$$\boxed{\mathbf{A}}\boxed{\mathbf{GT}}\boxed{\mathbf{G}}\boxed{\mathbf{TG}}\boxed{\mathbf{A}}\boxed{\mathbf{GT}}\boxed{\mathbf{G}}\boxed{\mathbf{TG}}\boxed{\mathbf{A}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{G}}\boxed{\mathbf{TG}}\boxed{\mathbf{A}}\boxed{\mathbf{GT}}\boxed{\mathbf{G}}\boxed{\mathbf{TG}}\boxed{\mathbf{A}}\boxed{\mathbf{GT}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_2.

Proposition 2.13. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_8 = \{e,o,r\} = \{ACC, GGT, CAG, CTG, CGA, TCG\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

$$\boxed{\mathbf{C}}\boxed{\mathbf{GT}}\boxed{\mathbf{G}}\boxed{\mathbf{TG}}\boxed{\mathbf{C}}\boxed{\mathbf{GT}}\boxed{\mathbf{G}}\boxed{\mathbf{TG}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{G}}\boxed{\mathbf{CA}}\boxed{\mathbf{C}}\boxed{\mathbf{AC}}\boxed{\mathbf{G}}\boxed{\mathbf{CA}}\boxed{\mathbf{C}}\boxed{\mathbf{AC}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_2.

Proposition 2.14. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern β_9 = {f,i,m} $= \{ACG, CGT, AGC, GCT, CAA, TTG\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

$$\boxed{\mathbf{C}}\boxed{\mathbf{TG}}\boxed{\mathbf{T}}\boxed{\mathbf{GT}}\boxed{\mathbf{C}}\boxed{\mathbf{TG}}\boxed{\mathbf{T}}\boxed{\mathbf{GT}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{G}}\boxed{\mathbf{AC}}\boxed{\mathbf{A}}\boxed{\mathbf{CA}}\boxed{\mathbf{G}}\boxed{\mathbf{AC}}\boxed{\mathbf{A}}\boxed{\mathbf{CA}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_2.

Proposition 2.15. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern β_{10} = {f,i,u} $= \{ACG, CGT, AGC, GCT, GAA, TTC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

$$\boxed{\mathbf{G}}\boxed{\mathbf{TC}}\boxed{\mathbf{T}}\boxed{\mathbf{CT}}\boxed{\mathbf{G}}\boxed{\mathbf{TC}}\boxed{\mathbf{T}}\boxed{\mathbf{CT}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{C}}\boxed{\mathbf{AG}}\boxed{\mathbf{A}}\boxed{\mathbf{GA}}\boxed{\mathbf{C}}\boxed{\mathbf{AG}}\boxed{\mathbf{A}}\boxed{\mathbf{GA}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_2.

Proposition 2.16. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern β_{11} = {f,o,x} $= \{ACG, CGT, CAG, CTG, GCC, GGC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

$$\boxed{\mathbf{A}}\boxed{\mathbf{GC}}\boxed{\mathbf{G}}\boxed{\mathbf{CG}}\boxed{\mathbf{A}}\boxed{\mathbf{GC}}\boxed{\mathbf{G}}\boxed{\mathbf{CG}}\boxed{\mathbf{A}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{T}}\boxed{\mathbf{CG}}\boxed{\mathbf{C}}\boxed{\mathbf{GC}}\boxed{\mathbf{T}}\boxed{\mathbf{CG}}\boxed{\mathbf{C}}\boxed{\mathbf{GC}}\boxed{\mathbf{T}}$$

is a 5LDCN in X_2.

Proposition 2.17. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern β_{12} = {f,o,y} $= \{ACG, CGT, CAG, CTG, GGA, TCC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

C	**GA**	**G**	**AG**	**C**	**GA**	**G**	**AG**	**C**

is a 5LDCN in X_1, and its dual necklace

G	**CT**	**C**	**TC**	**G**	**CT**	**C**	**TC**	**G**

is a 5LDCN in X_2.

Proposition 2.18. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_{13} = \{g,k,m\} = \{ACT, AGT, ATC, GAT, CAA, TTG\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

A	**TG**	**T**	**GT**	**A**	**TG**	**T**	**GT**	**A**

is a 5LDCN in X_1, and its dual necklace

T	**AC**	**A**	**CA**	**T**	**AC**	**A**	**CA**	**T**

is a 5LDCN in X_2.

Proposition 2.19. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_{14} = \{g,k,z'\} = \{ACT, AGT, ATC, GAT, TAA, TTA\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

G	**TA**	**T**	**AT**	**G**	**TA**	**T**	**AT**	**G**

is a 5LDCN in X_1, and its dual necklace

C	**AT**	**A**	**TA**	**C**	**AT**	**A**	**TA**	**C**

is a 5LDCN in X_2.

Proposition 2.20. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_{15} = \{g,l,u\} = \{ACT, AGT, ATG, CAT, GAA, TTC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. If See proposition (2.6): here, similarly, we have that

A	**TC**	**T**	**CT**	**A**	**TC**	**T**	**CT**	**A**

is a 5LDCN in X_1, and its dual necklace

T	**AG**	**A**	**GA**	**T**	**AG**	**A**	**GA**	**T**

is a 5LDCN in X_2.

Proposition 2.21. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_{16} = \{g,l,z'\} = \{ACT, AGT, ATG, CAT, TAA, TTA\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

$$\boxed{\mathbf{C}}\boxed{\mathbf{TA}}\boxed{\mathbf{T}}\boxed{\mathbf{AT}}\boxed{\mathbf{C}}\boxed{\mathbf{TA}}\boxed{\mathbf{T}}\boxed{\mathbf{AT}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{G}}\boxed{\mathbf{AT}}\boxed{\mathbf{A}}\boxed{\mathbf{TA}}\boxed{\mathbf{G}}\boxed{\mathbf{AT}}\boxed{\mathbf{A}}\boxed{\mathbf{TA}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_2.

Proposition 2.22. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_{17} = \{i,p,v\} = \{AGC, GCT, CCA, TGG, GAC, GTC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

$$\boxed{\mathbf{C}}\boxed{\mathbf{AC}}\boxed{\mathbf{G}}\boxed{\mathbf{CA}}\boxed{\mathbf{C}}\boxed{\mathbf{AC}}\boxed{\mathbf{G}}\boxed{\mathbf{CA}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{G}}\boxed{\mathbf{TG}}\boxed{\mathbf{C}}\boxed{\mathbf{GT}}\boxed{\mathbf{G}}\boxed{\mathbf{TG}}\boxed{\mathbf{C}}\boxed{\mathbf{GT}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_2.

Proposition 2.23. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_{18} = \{i,q,v\} = \{AGC, GCT, CCG, CGG, GAC, GTC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

$$\boxed{\mathbf{A}}\boxed{\mathbf{CG}}\boxed{\mathbf{C}}\boxed{\mathbf{GC}}\boxed{\mathbf{A}}\boxed{\mathbf{CG}}\boxed{\mathbf{C}}\boxed{\mathbf{GC}}\boxed{\mathbf{A}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{T}}\boxed{\mathbf{GC}}\boxed{\mathbf{G}}\boxed{\mathbf{CG}}\boxed{\mathbf{T}}\boxed{\mathbf{GC}}\boxed{\mathbf{G}}\boxed{\mathbf{CG}}\boxed{\mathbf{T}}$$

is a 5LDCN in X_2.

Proposition 2.24. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_{19} = \{j,k,z\} = \{AGG, CCT, ATC, GAT, GTA, TAC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

$$\boxed{\mathbf{A}}\boxed{\mathbf{CT}}\boxed{\mathbf{C}}\boxed{\mathbf{TC}}\boxed{\mathbf{A}}\boxed{\mathbf{CT}}\boxed{\mathbf{C}}\boxed{\mathbf{TC}}\boxed{\mathbf{A}}$$

is a 5LDCN in X_1, and its dual necklace

T GA G AG T GA G AG T

is a 5LDCN in X_2.

Proposition 2.25. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_{20} = \{j,v,w\} = \{AGG, CCT, GAC, GTC, GCA, TGC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

C TC G CT C TC G CT C

is a 5LDCN in X_1, and its dual necklace

G AG C GA G AG C GA G

is a 5LDCN in X_2.

Proposition 2.26. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_{21} = \{k,p,z\} = \{ATC, GAT, CCA, TGG, GTA, TAC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

C AC T CA C AC T CA C

is a 5LDCN in X_1, and its dual necklace

G TG A GT G TG A GT G

is a 5LDCN in X_2.

Proposition 2.27. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_{22} = \{l,s,y\} = \{ATG, CAT, CTA, TAG, GGA, TCC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

G AG T GA G AG T GA G

is a 5LDCN in X_1, and its dual necklace

C TC A CT C TC A CT C

is a 5LDCN in X_2.

Proposition 2.28. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_{23} = \{o,r,x\} = \{CAG, CTG, CGA, TCG, GCC, GGC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

$$\boxed{\mathbf{G}}\boxed{\mathbf{CG}}\boxed{\mathbf{T}}\boxed{\mathbf{GC}}\boxed{\mathbf{G}}\boxed{\mathbf{CG}}\boxed{\mathbf{T}}\boxed{\mathbf{GC}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{C}}\boxed{\mathbf{GC}}\boxed{\mathbf{A}}\boxed{\mathbf{CG}}\boxed{\mathbf{C}}\boxed{\mathbf{GC}}\boxed{\mathbf{A}}\boxed{\mathbf{CG}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_2.

Proposition 2.29. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the tern $\beta_{24} = \{q,v,w\} = \{CCG, CGG, GAC, GTC, GCA, TGC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.6): here, similarly, we have that

$$\boxed{\mathbf{C}}\boxed{\mathbf{GC}}\boxed{\mathbf{T}}\boxed{\mathbf{CG}}\boxed{\mathbf{C}}\boxed{\mathbf{GC}}\boxed{\mathbf{T}}\boxed{\mathbf{CG}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{G}}\boxed{\mathbf{CG}}\boxed{\mathbf{A}}\boxed{\mathbf{GC}}\boxed{\mathbf{G}}\boxed{\mathbf{CG}}\boxed{\mathbf{A}}\boxed{\mathbf{GC}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_2.

Proposition 2.30. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern $\gamma_1 = \{a,h,j,r\} = \{AAC, GTT, AGA, TCT, AGG, CCT, CGA, TCG\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. If $\gamma_1 = \{a,h,j,r\} = \{AAC, GTT, AGA, TCT, AGG, CCT, CGA, TCG\} \subset X_0$ then

$$\mathcal{P}(\{AAC, GTT, AGA, TCT, AGG, CCT, CGA, TCG\}) =$$

$$= \{ACA, TTG, GAA, CTT, GGA, CTC, GAC, CGT\} \subset X_1,$$

and we have that

$$\boxed{\mathbf{C}}\boxed{\mathbf{TT}}\boxed{\mathbf{G}}\boxed{\mathbf{GA}}\boxed{\mathbf{C}}\boxed{\mathbf{TT}}\boxed{\mathbf{G}}\boxed{\mathbf{GA}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_1,
or, shifting,

$$\boxed{\mathbf{G}}\boxed{\mathbf{GA}}\boxed{\mathbf{C}}\boxed{\mathbf{TT}}\boxed{\mathbf{G}}\boxed{\mathbf{GA}}\boxed{\mathbf{C}}\boxed{\mathbf{TT}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_1, so that X_1 is not a circular code;
we have also that

$$\mathcal{P}^2(\{AAC, GTT, AGA, TCT, AGG, CCT, CGA, TCG\}) =$$

$$= \{CAA, TGT, AAG, TTC, GAG, TCC, ACG, GTC\} \subset X_2,$$

so that the dual of the first necklace above (that is the necklace to which the complementarity map $\mathcal{C}$ is applied)

$$\boxed{\mathbf{G}}\boxed{\mathbf{TC}}\boxed{\mathbf{C}}\boxed{\mathbf{AA}}\boxed{\mathbf{G}}\boxed{\mathbf{TC}}\boxed{\mathbf{C}}\boxed{\mathbf{AA}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_2,
or, similarly, shifting,

$$\boxed{\mathbf{C}}\boxed{\mathbf{AA}}\boxed{\mathbf{G}}\boxed{\mathbf{TC}}\boxed{\mathbf{C}}\boxed{\mathbf{AA}}\boxed{\mathbf{G}}\boxed{\mathbf{TC}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_2, and we conclude that also X_2 is not a circular code.

Proposition 2.31. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern* $\gamma_2 = \{a,h,n,w\} = \{AAC, GTT, AGA, TCT, CAC, GTG, GCA, TGC\}$ *then neither* $X_1 = \mathcal{P}(X_0)$ *nor* $X_2 = \mathcal{P}^2(X_0)$ *are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

$$\boxed{\mathbf{G}}\boxed{\mathbf{CT}}\boxed{\mathbf{T}}\boxed{\mathbf{TG}}\boxed{\mathbf{G}}\boxed{\mathbf{CT}}\boxed{\mathbf{T}}\boxed{\mathbf{TG}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{C}}\boxed{\mathbf{CA}}\boxed{\mathbf{A}}\boxed{\mathbf{AG}}\boxed{\mathbf{C}}\boxed{\mathbf{CA}}\boxed{\mathbf{A}}\boxed{\mathbf{AG}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_2.

Proposition 2.32. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern* $\gamma_3 = \{a,h,n,z'\} = \{AAC, GTT, AGA, TCT, CAC, GTG, TAA, TTA\}$ *then neither* $X_1 = \mathcal{P}(X_0)$ *nor* $X_2 = \mathcal{P}^2(X_0)$ *are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

$$\boxed{\mathbf{G}}\boxed{\mathbf{AA}}\boxed{\mathbf{T}}\boxed{\mathbf{TG}}\boxed{\mathbf{G}}\boxed{\mathbf{AA}}\boxed{\mathbf{T}}\boxed{\mathbf{TG}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{C}}\boxed{\mathbf{CA}}\boxed{\mathbf{A}}\boxed{\mathbf{TT}}\boxed{\mathbf{C}}\boxed{\mathbf{CA}}\boxed{\mathbf{A}}\boxed{\mathbf{TT}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_2.

Proposition 2.33. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern* $\gamma_4 = \{a,l,n,y\} = \{AAC, GTT, ATG, CAT, CAC, GTG\}$ *then neither* $X_1 = \mathcal{P}(X_0)$ *nor* $X_2 = \mathcal{P}^2(X_0)$ *are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

$$\boxed{\mathbf{A}}\boxed{\mathbf{CC}}\boxed{\mathbf{T}}\boxed{\mathbf{TG}}\boxed{\mathbf{A}}\boxed{\mathbf{CC}}\boxed{\mathbf{T}}\boxed{\mathbf{TG}}\boxed{\mathbf{A}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{T}}\boxed{\mathbf{CA}}\boxed{\mathbf{A}}\boxed{\mathbf{GG}}\boxed{\mathbf{T}}\boxed{\mathbf{CA}}\boxed{\mathbf{A}}\boxed{\mathbf{GG}}\boxed{\mathbf{T}}$$

is a 5LDCN in X_2.

Proposition 2.34. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern* $\gamma_5 = \{b,d,e,w\} = \{AAG, CTT, ACA, TGT, ACC, GGT, GCA, TGC\}$ *then neither* $X_1 = \mathcal{P}(X_0)$ *nor* $X_2 = \mathcal{P}^2(X_0)$ *are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

$$\boxed{\mathbf{C}}\boxed{\mathbf{CA}}\boxed{\mathbf{G}}\boxed{\mathbf{TT}}\boxed{\mathbf{C}}\boxed{\mathbf{CA}}\boxed{\mathbf{G}}\boxed{\mathbf{TT}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{G}}\boxed{\mathbf{AA}}\boxed{\mathbf{C}}\boxed{\mathbf{TG}}\boxed{\mathbf{G}}\boxed{\mathbf{AA}}\boxed{\mathbf{C}}\boxed{\mathbf{TG}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_2.

Proposition 2.35. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern* $\gamma_6 = \{b,d,r,t\} = \{AAG, CTT, ACA, TGT, CGA, TCG, CTC, GAG\}$ *then neither* $X_1 = \mathcal{P}(X_0)$ *nor* $X_2 = \mathcal{P}^2(X_0)$ *are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

$$\boxed{\mathbf{C}}\boxed{\mathbf{GT}}\boxed{\mathbf{T}}\boxed{\mathbf{TC}}\boxed{\mathbf{C}}\boxed{\mathbf{GT}}\boxed{\mathbf{T}}\boxed{\mathbf{TC}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{G}}\boxed{\mathbf{GA}}\boxed{\mathbf{A}}\boxed{\mathbf{AC}}\boxed{\mathbf{G}}\boxed{\mathbf{GA}}\boxed{\mathbf{A}}\boxed{\mathbf{AC}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_2.

Proposition 2.36. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern* $\gamma_7 = \{b,d,t,z'\} = \{AAG, CTT, ACA, TGT, CTC, GAG, TAA, TTA\}$ *then neither* $X_1 = \mathcal{P}(X_0)$ *nor* $X_2 = \mathcal{P}^2(X_0)$ *are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

$$\boxed{\mathbf{C}}\boxed{\mathbf{AA}}\boxed{\mathbf{T}}\boxed{\mathbf{TC}}\boxed{\mathbf{C}}\boxed{\mathbf{AA}}\boxed{\mathbf{T}}\boxed{\mathbf{TC}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{G}}\boxed{\mathbf{GA}}\boxed{\mathbf{A}}\boxed{\mathbf{TT}}\boxed{\mathbf{G}}\boxed{\mathbf{GA}}\boxed{\mathbf{A}}\boxed{\mathbf{TT}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_2.

Proposition 2.37. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern* $\gamma_8 = \{b,k,p,t\} = \{AAG, CTT, ATC, GAT, CCA, TGG, CTC, GAG\}$ *then neither* $X_1 = \mathcal{P}(X_0)$ *nor* $X_2 = \mathcal{P}^2(X_0)$ *are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

$$\boxed{\mathbf{A}}\boxed{\mathbf{GG}}\boxed{\mathbf{T}}\boxed{\mathbf{TC}}\boxed{\mathbf{A}}\boxed{\mathbf{GG}}\boxed{\mathbf{T}}\boxed{\mathbf{TC}}\boxed{\mathbf{A}}$$

is a 5LDCN in X_1, and its dual necklace

T	GA	A	CC	T	GA	A	CC	T

is a 5LDCN in X_2.

Proposition 2.38. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern $\gamma_9 = \{c,d,t,u\} = \{AAT, ATT, ACA, TGT, CTC, GAG, GAA, TTC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

A	AG	G	TT	A	AG	G	TT	A

is a 5LDCN in X_1, and its dual necklace

T	AA	C	CT	T	AA	C	CT	T

is a 5LDCN in X_2.

Proposition 2.39. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern $\gamma_{10} = \{c,h,m,n\} = \{AAT, ATT, AGA, TCT, CAA, TTG, CAC, GTG\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

A	AC	C	TT	A	AC	C	TT	A

is a 5LDCN in X_1, and its dual necklace

T	AA	G	GT	T	AA	G	GT	T

is a 5LDCN in X_2.

Proposition 2.40. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern $\gamma_{11} = \{d,e,l,t\} = \{ACA, TGT, ACC, GGT, ATG, CAT, CTC, GAG\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

A	TC	C	CA	A	TC	C	CA	A

is a 5LDCN in X_1, and its dual necklace

T	TG	G	GA	T	TG	G	GA	T

is a 5LDCN in X_2.

Proposition 2.41. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern $\gamma_{12} = \{d,e,q,t\} = \{ACA, TGT, ACC, GGT, CCG, CGG, CTC, GAG\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

$$\boxed{\text{A}}\,\boxed{\text{GG}}\,\boxed{\text{C}}\,\boxed{\text{CA}}\,\boxed{\text{A}}\,\boxed{\text{GG}}\,\boxed{\text{C}}\,\boxed{\text{CA}}\,\boxed{\text{A}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\text{T}}\,\boxed{\text{TG}}\,\boxed{\text{G}}\,\boxed{\text{CC}}\,\boxed{\text{T}}\,\boxed{\text{TG}}\,\boxed{\text{G}}\,\boxed{\text{CC}}\,\boxed{\text{T}}$$

is a 5LDCN in X_2.

Proposition 2.42. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern $\gamma_{13} = \{d,f,p,u\} = \{ACA, TGT, ACG, CGT, CCA, TGG, GAA, TTC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

$$\boxed{\text{C}}\,\boxed{\text{AA}}\,\boxed{\text{G}}\,\boxed{\text{GT}}\,\boxed{\text{C}}\,\boxed{\text{AA}}\,\boxed{\text{G}}\,\boxed{\text{GT}}\,\boxed{\text{C}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\text{G}}\,\boxed{\text{AC}}\,\boxed{\text{C}}\,\boxed{\text{TT}}\,\boxed{\text{G}}\,\boxed{\text{AC}}\,\boxed{\text{C}}\,\boxed{\text{TT}}\,\boxed{\text{G}}$$

is a 5LDCN in X_2.

Proposition 2.43. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern $\gamma_{14} = \{d,i,t,u\} = \{ACA, TGT, AGC, GCT, CTC, GAG, GAA, TTC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

$$\boxed{\text{A}}\,\boxed{\text{AG}}\,\boxed{\text{G}}\,\boxed{\text{CA}}\,\boxed{\text{A}}\,\boxed{\text{AG}}\,\boxed{\text{G}}\,\boxed{\text{CA}}\,\boxed{\text{A}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\text{T}}\,\boxed{\text{TG}}\,\boxed{\text{C}}\,\boxed{\text{CT}}\,\boxed{\text{T}}\,\boxed{\text{TG}}\,\boxed{\text{C}}\,\boxed{\text{CT}}\,\boxed{\text{T}}$$

is a 5LDCN in X_2.

Proposition 2.44. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern $\gamma_{15} = \{d,p,t,x\} = \{ACA, TGT, CCA, TGG, CTC, GAG, GCC, GGC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

$$\boxed{\text{G}}\,\boxed{\text{GT}}\,\boxed{\text{T}}\,\boxed{\text{CC}}\,\boxed{\text{G}}\,\boxed{\text{GT}}\,\boxed{\text{T}}\,\boxed{\text{CC}}\,\boxed{\text{G}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\text{C}}\,\boxed{\text{GG}}\,\boxed{\text{A}}\,\boxed{\text{AC}}\,\boxed{\text{C}}\,\boxed{\text{GG}}\,\boxed{\text{A}}\,\boxed{\text{AC}}\,\boxed{\text{C}}$$

is a 5LDCN in X_2.

Proposition 2.45. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern $\gamma_{16} = \{d,p,t,z\} = \{ACA, TGT, CCA, TGG, CTC, GAG, GTA, TAC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

$$\boxed{\mathbf{G}}\boxed{\mathbf{GT}}\boxed{\mathbf{T}}\boxed{\mathbf{AG}}\boxed{\mathbf{G}}\boxed{\mathbf{GT}}\boxed{\mathbf{T}}\boxed{\mathbf{AG}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{C}}\boxed{\mathbf{CT}}\boxed{\mathbf{A}}\boxed{\mathbf{AC}}\boxed{\mathbf{C}}\boxed{\mathbf{CT}}\boxed{\mathbf{A}}\boxed{\mathbf{AC}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_2.

Proposition 2.46. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern $\gamma_{17} = \{e,s,t,u\} = \{ACC, GGT, CTA, TAG, CTC, GAG, GAA, TTC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

$$\boxed{\mathbf{A}}\boxed{\mathbf{AG}}\boxed{\mathbf{T}}\boxed{\mathbf{CC}}\boxed{\mathbf{A}}\boxed{\mathbf{AG}}\boxed{\mathbf{T}}\boxed{\mathbf{CC}}\boxed{\mathbf{A}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{T}}\boxed{\mathbf{GG}}\boxed{\mathbf{A}}\boxed{\mathbf{CT}}\boxed{\mathbf{T}}\boxed{\mathbf{GG}}\boxed{\mathbf{A}}\boxed{\mathbf{CT}}\boxed{\mathbf{T}}$$

is a 5LDCN in X_2.

Proposition 2.47. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern $\gamma_{18} = \{f,h,m,n\} = \{ACG, CGT, AGA, TCT, CAA, TTG, CAC, GTG\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

$$\boxed{\mathbf{A}}\boxed{\mathbf{AC}}\boxed{\mathbf{C}}\boxed{\mathbf{GA}}\boxed{\mathbf{A}}\boxed{\mathbf{AC}}\boxed{\mathbf{C}}\boxed{\mathbf{GA}}\boxed{\mathbf{A}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{T}}\boxed{\mathbf{TC}}\boxed{\mathbf{G}}\boxed{\mathbf{GT}}\boxed{\mathbf{T}}\boxed{\mathbf{TC}}\boxed{\mathbf{G}}\boxed{\mathbf{GT}}\boxed{\mathbf{T}}$$

is a 5LDCN in X_2.

Proposition 2.48. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern $\gamma_{19} = \{h,j,k,n\} = \{AGA, TCT, AGG, CCT, ATC, GAT, CAC, GTG\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

$$\boxed{\mathbf{A}}\boxed{\mathbf{TG}}\boxed{\mathbf{G}}\boxed{\mathbf{GA}}\boxed{\mathbf{A}}\boxed{\mathbf{TG}}\boxed{\mathbf{G}}\boxed{\mathbf{GA}}\boxed{\mathbf{A}}$$

is a 5LDCN in X_1, and its dual necklace

T | TC | C | CA | T | TC | C | CA | T

is a 5LDCN in X_2.

Proposition 2.49. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern $\gamma_{20} = \{h,j,n,x\} = \{AGA, TCT, AGG, CCT, CAC, GTG, GCC, GGC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

A | CC | G | GA | A | CC | G | GA | A

is a 5LDCN in X_1, and its dual necklace

T | TC | C | GG | T | TC | C | GG | T

is a 5LDCN in X_2.

Proposition 2.50. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern $\gamma_{21} = \{h,i,m,y\} = \{AGA, TCT, AGC, GCT, CAA, TTG, GGA, TCC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

C | CT | G | AA | C | CT | G | AA | C

is a 5LDCN in X_1, and its dual necklace

G | TT | C | AG | G | TT | C | AG | G

is a 5LDCN in X_2.

Proposition 2.51. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern $\gamma_{22} = \{h,n,q,y\} = \{AGA, TCT, CAC, GTG, CCG, CGG, GGA, TCC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

C | CT | T | GG | C | CT | T | GG | C

is a 5LDCN in X_1, and its dual necklace

G | CC | A | AG | G | CC | A | AG | G

is a 5LDCN in X_2.

Proposition 2.52. *If a maximal non C^3 self-complementary code X_0 of the 312 in the list contains the quatern $\gamma_{23} = \{h,n,s,y\} = \{AGA, TCT, CAC, GTG, CTA, TAG, GGA, TCC\}$ then neither $X_1 = \mathcal{P}(X_0)$ nor $X_2 = \mathcal{P}^2(X_0)$ are circular codes.*

Proof. See proposition (2.30): here, similarly, we have that

20.

$$\boxed{\mathbf{C}}\boxed{\mathbf{CT}}\boxed{\mathbf{T}}\boxed{\mathbf{AC}}\boxed{\mathbf{C}}\boxed{\mathbf{CT}}\boxed{\mathbf{T}}\boxed{\mathbf{AC}}\boxed{\mathbf{C}}$$

is a 5LDCN in X_1, and its dual necklace

$$\boxed{\mathbf{G}}\boxed{\mathbf{GT}}\boxed{\mathbf{A}}\boxed{\mathbf{AG}}\boxed{\mathbf{G}}\boxed{\mathbf{GT}}\boxed{\mathbf{A}}\boxed{\mathbf{AG}}\boxed{\mathbf{G}}$$

is a 5LDCN in X_2.

We now report a list of the couples, the terns and the quaterns named above and then a table of their occurences in the 312 maximal non C^3 self-complementary codes, numbered in the order refering to [10]. We write only an occurence for each code.

$$\alpha_1 = \{a,p\}, \alpha_2 = \{b,y\}, \alpha_3 = \{e,m\}, \alpha_4 = \{j,u\},$$

$$\beta_1 = \{a,r,w\}, \beta_2 = \{a,s,z''\}, \beta_3 = \{b,r,w\}, \beta_4 = \{b,z,z''\}, \beta_5 = \{c,s,z''\}, \beta_6 = \{c,z,z''\},$$

$$\beta_7 = \{e,l,s\}, \beta_8 = \{e,o,r\}, \beta_9 = \{f,i,m\}, \beta_{10} = \{f,i,u\}, \beta_{11} = \{f,o,x\}, \beta_{12} = \{f,o,y\},$$

$$\beta_{13} = \{g,k,m\}, \beta_{14} = \{g,k,z'\}, \beta_{15} = \{g,l,u\}, \beta_{16} = \{g,l,z'\}, \beta_{17} = \{i,p,v\}, \beta_{18} = \{i,q,v\},$$

$$\beta_{19} = \{j,k,z\}, \beta_{20} = \{j,v,w\}, \beta_{21} = \{k,p,z\}, \beta_{22} = \{l,s,y\}, \beta_{23} = \{o,r,x\}, \beta_{24} = \{q,v,w\},$$

$$\gamma_1 = \{a,h,j,r\}, \gamma_2 = \{a,h,n,w\}, \gamma_3 = \{a,h,n,z'\}, \gamma_4 = \{a,l,n,y\},$$

$$\gamma_5 = \{b,d,e,w\}, \gamma_6 = \{b,d,r,t\}, \gamma_7 = \{b,d,t,z'\}, \gamma_8 = \{b,k,p,t\},$$

$$\gamma_9 = \{c,d,t,u\}, \gamma_{10} = \{c,h,m,n\}, \gamma_{11} = \{d,e,l,t\}, \gamma_{12} = \{d,e,q,t\},$$

$$\gamma_{13} = \{d,f,p,u\}, \gamma_{14} = \{d,i,t,u\}, \gamma_{15} = \{d,p,t,x\}, \gamma_{16} = \{d,p,t,z\},$$

$$\gamma_{17} = \{e,s,t,u\}, \gamma_{18} = \{f,h,m,n\}, \gamma_{19} = \{h,j,k,n\}, \gamma_{20} = \{h,j,n,x\},$$

$$\gamma_{21} = \{h,i,m,y\}, \gamma_{22} = \{h,n,q,y\}, \gamma_{23} = \{h,n,s,y\}.$$

abcegikvxy	α_2	*abcegivxyz”*	α_2	*acfghjopqz”*	α_1	*bcdegivxyz”*	α_2	*cdfglopqtu*	β_{15}	*dgkpuvwxyz’*	β_{14}
abcfgjlopq	α_1	*abcegkovxy*	α_2	*acfghlnoqy*	β_{12}	*bcdfgikptx*	γ_8	*cdflopqstu*	γ_9	*dhjpvwxzz’z”*	β_{20}
abcflopqst	α_1	*abceikvxyz*	α_2	*acghijknvx*	γ_{19}	*bcdfopqstz”*	β_5	*cdghijkpvx*	β_{17}	*djkpuvwxz z’*	α_4
abciknvxyz	α_2	*abceivxyzz”*	α_2	*acghijkpvx*	α_1	*bcdgikptvx*	β_{17}	*cdghijpvxz”*	β_{17}	*djopqrsuz’z”*	α_4
abcknovxyz	α_2	*abcekovxyz*	α_2	*acghijlnqv*	β_{18}	*bcefgijlmq*	α_3	*cdgijkpuvx*	α_4	*djpqsuvwz’z”*	α_4
abclopqstv	α_1	*abcfgijlpq*	α_1	*acghijlpqv*	α_1	*bcefglmoqt*	α_3	*cdgikptuvx*	β_{17}	*djpuvwxzz’z”*	α_4
acegijkuvx	α_4	*abcfgijpqz”*	α_1	*acghijpvxz”*	α_1	*bcefglmoqy*	α_2	*cefghijkmx*	α_3	*dkpuvwxyzz’*	β_{21}
acfghijlpq	α_1	*abcfgjopqz”*	α_1	*acgijknuvx*	α_4	*bcefjlmoqs*	α_3	*cefghikmxy*	α_3	*dpqstuvwz’z”*	β_{24}
acfghijpqz”	α_1	*abcfglnoqy*	α_2	*acgijkpuvx*	α_1	*bceflmoqsy*	α_2	*cefghilmqy*	α_3	*dptuvwxzz’z”*	γ_{15}
achijknvxz	β_{19}	*abcfglopqt*	α_1	*acgikptuvx*	α_1	*acikptuvxz*	α_1	*cefghjlmoq*	α_3	*eghlmoqry z’*	α_3
acijknuvxz	α_4	*abcfjlopqs*	α_1	*achijkpvxz*	α_1	*bcfglmnoqy*	α_2	*cefghlmoqy*	α_3	*eghmoqryz’z”*	α_3
ahnrwxyzz’z”	β_1	*abcfjopqsz”*	α_1	*achijnvxzz”*	β_6	*bcflmnoqsy*	α_2	*cefgijkmux*	α_3	*eghmvwxyz’z”*	α_3
ahprwxyzz’z”	α_1	*abcflnoqsy*	α_2	*achijpvxzz”*	α_1	*bcgijlmnqv*	β_{18}	*cefgijlmqu*	α_3	*egkmtuvwxz’*	α_3
akptuvwxzz’	α_1	*abcfopqstz”*	α_1	*achinvxyzz”*	β_6	*bdektvwxzz’*	γ_5	*cefgikmtux*	α_3	*egkmuvwxyz’*	α_3
apuvwxyzz’z”	α_1	*abcgijlpqv*	α_1	*acijkpuvxz*	α_1	*bdeoqrstz’z”*	β_8	*cefgjlmoqu*	α_3	*egmruwxyz’z”*	α_3
bcdefgikxy	α_2	*abcgiknvxy*	α_2	*bcfgijlmnq*	β_9	*bdeoqrsyz’z”*	α_2	*cefgkmotux*	α_3	*egmuvwxyz’z”*	α_3
bcdefgixyz”	α_2	*abcgikptvx*	α_1	*aehrwxyzz’z”*	β_1	*bdeqrstwz’z”*	β_3	*cefglmoqtu*	α_3	*ehlmoqrsyz’*	α_3
bcdefloqst	β_7	*abcgjlopqv*	α_1	*ahjlnoqrsz’*	γ_1	*bdeqrswyz’z”*	α_2	*ceflmoqstu*	α_3	*ehmoqrsyz’z”*	α_3
bcefgjlmoq	α_3	*abcgknovxy*	α_2	*ahjnoqrsz’z”*	β_2	*bderwxyzz’z”*	α_2	*ceghijkmvx*	α_3	*ehmorxyzz’z”*	α_3
bceflmoqst	α_3	*abcglopqtv*	α_1	*ahjnrwxzz’z”*	β_1	*ahlnoqrsyz’*	β_{22}	*ceghikmvxy*	α_3	*ehmvwxyzz’z”*	α_3
bdpqrstwz’z”	β_3	*abcijknvxz*	β_{19}	*ahjnvwxzz’z”*	β_{20}	*bdevwxyzz’z”*	α_2	*cegijkmuvx*	α_3	*ekmouvxyzz’*	α_3
bdpqrswyz’z”	α_2	*abcikptvxz*	α_1	*bdetvwxzz’z”*	β_4	*bdjpqrswz’z”*	β_3	*cegikmtuvx*	α_3	*ekmuvwxyzz’*	α_3
blmnoqrsyz’	α_2	*abcinvxyzz”*	α_2	*ahjpqrswz’z”*	α_1	*bdkptvwxzz’*	β_{21}	*cegkmotuvx*	α_3	*elmoqrstuz’*	α_3
bmopqrsyz’z”	α_2	*abcjlopqsv*	α_1	*ahjprwxzz’z”*	α_1	*bdopqrstz’z”*	γ_6	*cfghijlmpq*	β_9	*emoqrstuz’z”*	α_3
cdefgijkux	α_4	*abkptvwxzz’*	α_1	*ahjpvwxzz’z”*	α_1	*bdopqrsyz’z”*	α_2	*cfghilmnqy*	β_9	*emoruxyzz’z”*	α_3
cdefgiktux	β_{10}	*ablnoqrsyz’*	α_2	*ahjopqrsz’z”*	α_1	*bdpqstvwz’z”*	β_{24}	*cfghjlmnoq*	γ_{10}	*emotuvxzz’z”*	α_3
cefghijlmq	α_3	*abnoqrstz’z”*	β_2	*ahnoqrsyz’z”*	β_2	*bdprwxyzz’z”*	α_2	*cfghlmnoqy*	β_{12}	*emouvxyzz’z”*	α_3
cfghijlmnq	β_9	*abnoqrsyz’z”*	α_2	*ahnorxyzz’z”*	β_{23}	*bdptvwxzz’z”*	β_4	*cfgijlmnqu*	α_4	*emruwxyzz’z”*	α_3
dgjpqruwz’z”	α_4	*abnoqstvz’z”*	β_2	*ahnvwxyzz’z”*	γ_2	*bdpvwxyzz’z”*	α_2	*cfgjlmnoqu*	α_4	*emtuvwxzz’z”*	α_3
djpqrsuwz’z”	α_4	*abnorxyzz’z”*	α_2	*ahopqrsyz’z”*	α_1	*belmoqrstz’*	α_3	*cfglmnoqtu*	β_{15}	*ghjlmnoqrz’*	β_{16}
dkptuvwxzz’	β_{21}	*abnotvxzz’z”*	β_4	*ahpqrswyz’z”*	α_1	*belmoqrsyz’*	α_2	*cghijkmnvx*	β_{13}	*ghlmnoqryz’*	β_{16}
eghmrwxyz’z”	α_3	*abnovxyzz’z”*	α_2	*ahpvwxyzz’z”*	α_1	*bemoqrstzz”*	α_3	*cghijlmnqv*	β_{18}	*ghlmopqryz’*	β_{16}
ehmrwxyzz’z”	α_3	*abnrwxyzz’z”*	α_2	*ajknuvwxzz’*	α_4	*bemoqrsyz’z”*	α_2	*cghikmnvxy*	β_{13}	*gjlmnoqruz’*	α_4
ekmotuvxzz”	α_3	*abntvwxzz’z”*	β_4	*ajkpuvwxzz’*	α_1	*blmopqrsyz’*	α_2	*cgijkmnuvx*	α_4	*gjlmopqruz’*	α_4
ekmtuvwxzz’	α_3	*abnvwxyzz’z”*	α_2	*ajnuvwxzz’z”*	α_4	*bmnoqrsyz’z”*	α_2	*cgijlmnquv*	α_4	*gjmopqruz’z”*	α_4
emuvwxyzz’ z”	α_3	*abopqrstz’z”*	α_1	*ajpuvwxzz’z”*	α_1	*bmpqrstwz’z”*	β_3	*cgikmntuvx*	β_{13}	*gjmpqruwz’z”*	α_4
hlmnoqrsyz’	β_{22}	*abopqrsyz’z”*	α_1	*akpuvwxyzz’*	α_1	*bmpqrswyz’z”*	α_2	*cgjlmnoquv*	α_4	*hjmnrwxzz’z”*	γ_{20}
jlmnoqrsuz’	α_4	*abopqstvz’z”*	α_1	*anoruxyzz’z”*	β_{23}	*bmpqstvwz’z”*	β_{24}	*cgkmnotuvx*	β_{13}	*hjmnvwxzz’z”*	β_{20}
jlmnoqsuvz’	α_4	*abpqrstwz’z”*	α_1	*anruwxyzz’z”*	β_1	*cdefghloqy*	β_{12}	*cglmnoqtuv*	β_{15}	*hlmopqrsyz’*	β_{22}
jmopqrsuz’z”	α_4	*abpqrswyz’z”*	α_1	*apruwxyzz’z”*	α_1	*cdefghoqyz”*	β_{12}	*chijkmnvxz*	β_{19}	*hmnoqrsyz’z”*	γ_{22}
abcefgkotx	β_{11}	*abpqstvwz’z”*	α_1	*aptuvwxzz’z”*	α_1	*cdefgijlqu*	α_4	*chikmnvxyz*	γ_{10}	*hmnorxyzz’z”*	β_{23}
abcgijlnqv	β_{18}	*abprwxyzz’z”*	α_1	*bcdefgkotx*	β_{11}	*cdefgikuxy*	β_{10}	*cijkmnuvxz*	α_4	*jkmnuvwxzz’*	α_4
acehivxyzz”	β_6	*abptvwxzz’z”*	α_1	*bcdefgkoxy*	α_2	*cdefgjloqu*	α_4	*deghoqryz’z”*	β_8	*jlmopqrsuz’*	α_4
bcdfjopqsz”	β_5	*abpvwxyzz’z”*	α_1	*bcdefgloqt*	γ_{11}	*cdefgkotux*	β_{11}	*degktuvwxz’*	β_{14}	*jlmopqsuvz’*	α_4
degkuvwxyz’	β_{14}	*acefgijkux*	α_4	*bcdefgloqy*	α_2	*cdefgloqtu*	β_{15}	*dehoqrsyz’z”*	β_8	*jmnoqrsuz’z”*	α_4
ghjlmopqrz’	β_{16}	*acefgiktux*	β_{10}	*bcdefgoqyz”*	β_{12}	*cdefloqstu*	β_7	*deoqrstuz’z”*	β_8	*jmnoqsuvz’z”*	α_4
mnoruxyzz’z”	β_{23}	*acefgkotux*	β_{11}	*bcdefjloqs*	β_7	*cdegijkuvx*	α_4	*deqrstuwz’z”*	γ_{12}	*jmnuvwxzz’z”*	α_4
mpqstuvwz’z”	β_{24}	*acehijkvxz*	β_{19}	*bcdefjoqsz”*	β_5	*cdegiktuvx*	γ_9	*dghjpvwxz’z”*	β_{20}	*jmopqsuvz’z”*	α_4
abcefgikxy	α_2	*acehijvxzz”*	β_6	*bcdefloqsy*	α_2	*cdfgijkpux*	α_4	*dgjkpuvwxz’*	α_4	*jmpqrsuwz’z”*	α_4
abcefgixyz”	α_2	*aceijkuvxz*	α_4	*bcdefoqstz”*	β_5	*cdfgijlpqu*	α_4	*dgjopqruz’z”*	α_4	*jmpqsuvwz’z”*	α_4
abcefgkoxy	α_2	*acfghilnqy*	γ_4	*bcdefoqsyz”*	α_2	*cdfgikptux*	β_{10}	*dgjpuvwxz’z”*	α_4	*kmptuvwxzz’*	β_{21}
abcefloqst	β_7	*acfghjlopq*	α_1	*bcdegikvxy*	α_2	*cdfgjlopqu*	α_4	*dgkptuvwxz’*	β_{14}	*lmnoqrsuyz’*	β_{22}

Table 2.Occurences of the “forbidden configurations“ in the 312 maximal non C^3 self-complementary codes.

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