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SEMANTIC WEB SEARCH BASED ON ROUGH SETS AND FUZZY FORMAL CONCEPT ANALYSIS

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Abstract

Fuzzy Formal Concept Analysis (FFCA) is a generalization of *Formal Concept Analysis* (FCA) for modeling uncertainty information. FFCA provides a mathematical framework which can support the construction of formal ontologies in the presence of uncertainty data for the development of the Semantic Web. In this paper, we show how rough set theory can be employed in combination with FFCA to perform Semantic Web search and discovery of information in the Web.

Key words: Semantic Web, Formal Concept Analysis, fuzzy information, rough set theory.

1. Introduction

Formal Concept Analysis (FCA) has been introduced by Wille in 1982 for analyzing and structuring a domain of interest [Wille(1982)]. FCA provides a mathematical framework which can support several activities, in different research fields, as for instance software engineering, requirements analysis, component retrieval, etc... [Tilley et al.(2003)]. It is also a well-founded methodological approach for the construction of ontologies for Semantic Web development [Lukasiewicz and Straccia(2001), Lukasiewicz and Straccia(2009), Wang and He(2006)]. In fact, FCA can serve as a guideline for ontology building because it allows the identification of concepts by factoring out their commonalities while preserving concept specialization relationships. Semi-automatic methods and interactive tools for the ontology developers have been proposed, as for instance [Bai and Liu(2008), Hwang et al.(2005), Tho et al.(2006)].

Fuzzy Formal Concept Analysis (FFCA) is a generalization of FCA for modeling uncertainty information [Belohlávek et al.(2008)]. FFCA can support ontology construction when some information is more relevant than other data, or Semantic Web search when the user is not sure about what he/she is looking for. However, often, in real life applications user needs cannot be described on the basis of formal concepts only [Saquer and Deogun(2001)], and "approximate concepts will become increasingly more important as Semantic Web becomes a reality" [Doherty et al.(2003)]. In this paper, we show how rough set theory [Pawlak(1982)] can be employed in combination with FFCA to perform Semantic Web search and discovery of information in the Web. According to this proposal, in the case the required data are not modeled by any formal concept, the user can search and discovery information in the Web that are closer to his/her preferences by following a twofold approach. Thanks to the notions of lower/upper approximations, the user can select super/subsets of the data (objects) he/she is looking for. Furthermore, the notion of a fuzzy context in FFCA allows the user to choose, within the selected sets, specific objects that, on the basis of "grades of membership", allow him/her to quantify "how much" they are described by the required attributes and, therefore, "how much" these objects correspond to the user needs.

The paper is organized as follows. In Sections 2 and 3 Formal Concept Analysis and Fuzzy Formal Concept Analysis are recalled, respectively. In Section 4, the relationship between FCA and rough set theory is given and, successively, in Subsection 4.1, Web search based on rough sets and FFCA is shown. Finally in Section 5, the Related Work is presented and Section 6 concludes.

2. FCA

In FCA a *context* is a triple (O,A,R), where O is a set of *objects*, A is a set of *attributes*, and R is a binary relation between O and A. For instance, consider a context named *Sardinia Hotels*, suppose that the set O is defined by the following six objects representing six different hotels:

$O = \{$H1, H2, H3, H4, H5, H6$\}$

and the set A is defined by six possible attributes of these objects:

$A = \{$Tennis, Theater, SwPool, Meal, Sea, Cinema$\}$

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where *SwPool* stands for swimming pool. Furthermore, suppose the hotels are related to the above attributes according to the binary relation R defined by the following table:

	Tennis	Theater	SwPool	Meal	Sea	Cinema
H1		x		x	x	
H2	x		x	x		x
H3		x		x	x	
H4	x		x		x	
H5				x	x	
H6	x		x		x	

Table 1: The *Sardinia Hotels* context in (non-fuzzy) FCA

According to Table 1, we say that, for instance, the hotel $H4$ has, or is described by, three attributes, namely $Tennis$, $SwPool$, and Sea, and viceversa, these three attributes apply to the object $H4$. Given a context, in FCA a concept is a pair (E,I), where the former element, referred to as concept *extent*, is a set consisting of precisely those objects having all the attributes from the latter and, conversely, the latter, referred to as concept *intent*, is a set containing precisely those attributes that apply to all the objects from the former. For instance, a concept of the *Sardinia Hotels* context is:

($(H4, H6)$,$(Tennis, SwPool, Sea)$)

since both $H4$ and $H6$ have the attributes $Tennis$, $SwPool$, and Sea, and viceversa, all these attributes apply to both the objects $H4,H6$.

Given two concepts of a context, (E_1,I_1), (E_2,I_2), it is possible to establish an *inheritance relation* ($\leq$) between them according to the following condition:

$(E_1,I_1) \leq (E_2,I_2)$ iff $E_1 \subseteq E_2$ (iff $I_2 \subseteq I_1$).

In particular, (E_1,I_1) is called *subconcept* of (E_2,I_2) and (E_2,I_2) is called *superconcept* of (E_1,I_1).

Given a context (O,A,R), consider the set of all the concepts of this context, indicated as $\mathcal{L}(O,A,R)$. Then:

$(\mathcal{L}(O,A,R),\leq)$

is a complete lattice called *Concept Lattice* (also referred to as *Galois Lattice*), i.e., for each subset of concepts, the greatest common subconcept and the least common superconcept exist [Wille(1982)]. For instance, the Concept Lattice that can be constructed from the context of Table 1 is shown in Figure 1. Note that nodes are labeled with the concepts of the context, and arcs are established among the nodes whose associated concepts are in $\leq$ relation. The Concept Lattice has two special nodes, the maximum and minimum nodes, grouping all the objects and the attributes of the context, respectively.

In Figure 1, the least common superconcept of, for instance, the concepts ($(H4,H6)$, $(Tennis, SwPool,Sea)$) and ($(H1,H3)$,$(Theater,Meal,Sea)$) is the concept ($(H1,H3,H4,H5,H6)$, (Sea)), having as set of attributes the intersection of the sets of attributes of the concepts. Whereas the greatest common subconcept of the concepts ($(H2,H4,H6)$,$(Tennis,SwPool)$)
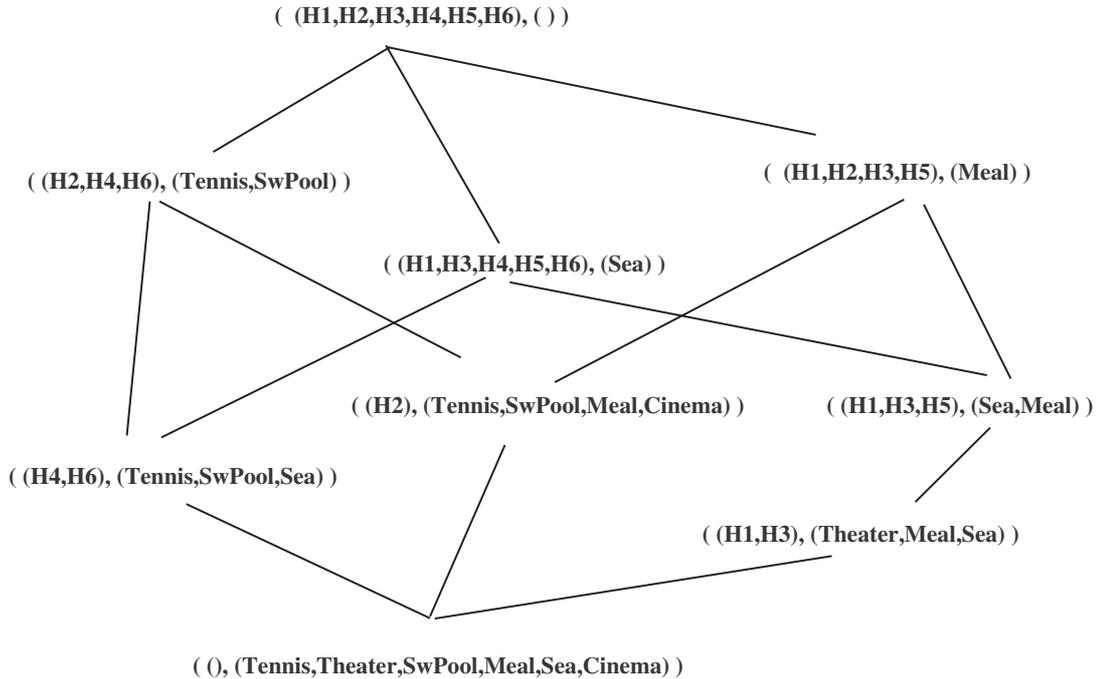
Figure 1: Concept Lattice of the *Sardinia Hotels* context

and ($(H1,H2,H3,H5),(Meal)$) is the concept ($(H2),(Tennis,SwPool,Meal, Cinema)$), having as set of objects the intersection of the sets of objects of the concepts.

Unfortunately, modeling a domain of interest with traditional FCA (i.e., with non-fuzzy sets) can be inaccurate when the attributes do not describe the objects in a uniform way or, in other words, a given attribute applies to different objects in different ways. For instance, in our example, consider the attribute *Sea*. One should be able to distinguish the hotels located just on the sea, from that having a walking distance seaside (reachable in, for instance, ten or twenty minutes). Analogously, regarding the attribute *Meal*, we would like to be aware about the hotels providing both lunch and dinner, rather than half-board. Without the introduction of fuzzy information, we have no way to specify how appropriate is a feature, or an attribute, to a given object, therefore describing all the objects in a uniform way.

3. Fuzzy FCA

A *fuzzy set A* in a space of points X is characterized by a *membership function* $\mu_A(x)$ which associates with each point x in X a real number in the interval [0,1] representing the *grade of membership* of x in A [Zadeh(1965)]. Note that for an ordinary set, the membership function can take only the values 1 and 0, depending on x does or does not belong to A, respectively. Just to provide an example, assume X is a set of people, a fuzzy set *Young* is defined by associating with each person in X a real number in [0,1] establishing the degree of youth of the person, such that the nearer this value to unity, the higher the grade of membership of a person in the set *Young*. The notion of a *fuzzy relation* can be obtained by generalizing the notion of a fuzzy set

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as follows. A fuzzy relation R in $X \times Y$, is a fuzzy set in the product space $X \times Y$.

For instance, consider the *Sardinia Hotels* context with fuzzy information. This context, referred to as *fuzzy context*, is specified by the fuzzy relation given in Table 2. In particular, crosses in Table 1 have been replaced by grades of membership, from 0 to 1, each allowing us to quantify "how much" an object has, or is described by, an attribute and viceversa an attribute applies to an object.

	Tennis	Theater	SwPool	Meal	Sea	Cinema
H1		0.7		1.0	1.0	
H2	0.6		1.0	0.5		0.9
H3		0.8		0.5	0.7	
H4	0.8		1.0		1.0	
H5				1.0	0.3	
H6	0.8		1.0		0.8	

Table 2: The *Sardinia Hotels* context in fuzzy FCA

For instance, consider the hotel $H2$ in Table 2. It has the attribute *SwPool* with grade of membership 1.0, which means that such attribute fully applies to the hotel $H2$ (and viceversa the hotel $H2$ can be properly described by the attribute *SwPool*). Instead, the object $H2$ has the attribute *Meal* with a membership value 0.5, which means that such an attribute partially applies to this hotel (for instance it could provide meals just for dinner). Analogously, in the case of $H3$, the value 0.7 in correspondence with the attribute *Sea* means that this feature better describes the hotels $H1$, $H4$ or $H6$ than $H3$, but it is more appropriate to $H3$ than $H5$ (having $H5$ a lower grade of membership with *Sea*, i.e., 0.3). In order to address only objects related to attributes with relevant grades of membership, a threshold is fixed such that the pairs with membership values less than the threshold are ignored. For instance, consider our running example and assume that a threshold is fixed equal to 0.5. The grade of membership 0.3 between $H5$ and *Sea* is ignored and treated analogously to the grades of membership that in Table 2 are not specified.

A fuzzy concept of the *Sardinia Hotels* fuzzy context is, for instance, the pair:

((($H1, 0.7$),($H3, 0.5$)), (*Theater*,*Meal*,*Sea*))

In fact the objects $H1$,$H3$ share the attributes *Theater*, *Meal* and *Sea* and, viceversa, these three attributes apply to the objects $H1$ and $H3$ with membership values which are not less than the threshold.

It is important to note that in a concept, in the case the attributes apply to an object with different grades of membership, the minimum among them is selected. For instance, the object $H3$ has the attributes *Theater*, *Meal* and *Sea* with different grades of membership, that are 0.8, 0.5 and 0.7, respectively. Therefore, in the concept above, the minimum value between them has been selected because it represents the highest common grade of membership that allows $H3$ to be described by the all the three attributes *Theater*, *Meal* and *Sea*.

Fuzzy Concept Lattices can be defined similar to Concept Lattices, on the basis of fuzzy set theory. Informally, given two fuzzy sets A, B, the fuzzy set intersection and fuzzy set union are defined by fuzzy sets where the membership functions are the minimum and maximum of the membership functions of A and B, respectively. For instance, the Fuzzy Concept Lattice that can be constructed from the context of Table 2 is shown in Figure 2. Analogously to Concept

(((H1,1.0),(H2,1.0),(H3,1.0),(H4,1.0),(H5,1.0),(H6,1.0)), ( ))

(((H2,0.6),(H4,0.8),(H6,0.8)), (Tennis,SwPool))

(((H1,1.0),(H2,0.5),(H3,0.5),(H5,1.0)),(Meal))

(((H1,1.0),(H3,0.7),(H4,1.0),(H6,0.8)), (Sea))

(((H2,0.5)), (Tennis,SwPool,Meal,Cinema))

(((H4,0.8),(H6,0.8)), (Tennis,SwPool,Sea))

(((H1,0.7),(H3,0.5)), (Theater,Meal,Sea))
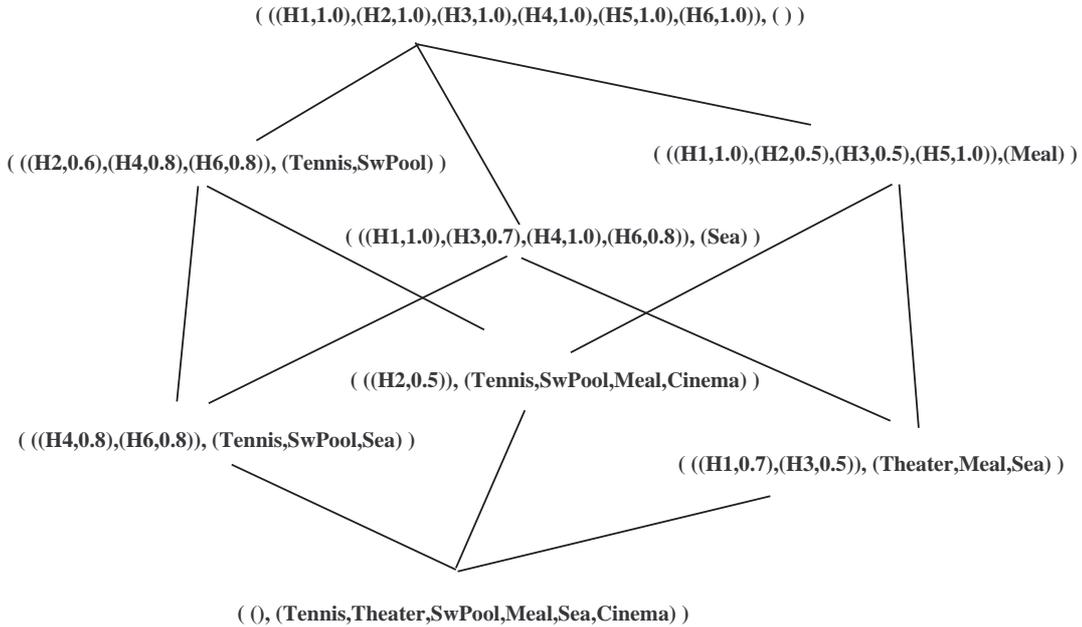
((), (Tennis,Theater,SwPool,Meal,Sea,Cinema))

Figure 2: Fuzzy Concept Lattice of the *Sardinia Hotels* context

Lattices, nodes are labeled with the fuzzy concepts of the context, and arcs are established among the nodes that are in inheritance relation. The Fuzzy Concept Lattice has two special nodes, the maximum and minimum nodes, grouping all the objects and the attributes of the context, respectively. In particular, the membership values associated with the objects of the maximum node are all equal to one. Also in Fuzzy Concept Lattices, for any subset of concepts, the greatest common subconcept and the least common superconcept are always defined. For instance, consider the concepts:

($((H2, 0.6),(H4, 0.8),(H6, 0.8))$, $(Tennis, SwPool)$)

($((H1, 1.0),(H3, 0.7),(H4, 1.0),(H6, 0.8))$, (Sea))

The greatest common subconcept is:

($((H4, 0.8),(H6, 0.8))$, $(Tennis, SwPool, Sea)$)

where the fuzzy set intersection of the objects has been performed. As mentioned above, fuzzy set intersection requires that the minimum among different grades of membership associated with the same object is selected (for instance, in the case of $H4$, 0.8).

4. Rough Sets and FCA

Rough set theory is an extension of classical set theory with two additional operators, namely approximation operators, originally introduced in [Pawlak(1982)]. Among the various formulations that can be found in the literature, below the one given in [Yao(1996)] is briefly recalled.

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Let U be a finite and non-empty universe of objects and E be an equivalence relation on U. E induces a partition of the universe U, indicated as U/E, and the pair $apr=(U,E)$ is referred to as an *approximation space*. An equivalence class in U/E is referred to as an *elementary set*. Any finite union of elementary sets is called a *definable set*. Given an arbitrary set $X \subseteq U$ it may not correspond to a definable set because X may include and exclude objects that belong to different definable sets. However, X can be approximated from below and above by a pair of definable sets referred to as the *lower* and *upper approximations* of X. Intuitively, the lower approximation is the greatest definable set contained in X and the upper approximation is the least definable set containing X.

The notion of approximation can be naturally introduced into FCA. It has been extensively investigated in the literature, see for instance [Saquer and Deogun(2001), Xu et al.(2008), Yao and Chen(2006)]. According to [Yao(1996), Yao and Chen(2006)], a Concept Lattice can be seen as the family of *all definable concepts*. Formal concepts that do not belong to a given Concept Lattice are called *non-definable concepts*. In particular, a formal concept consists of a *definable set of objects* and a *definable set of attributes*. Given a Concept Lattice, a set of objects (attributes) that is not the extension (intension) of any formal concept can be approximated by definable sets of objects (attributes) according to rough set theory.

In line with [Xu et al.(2008), Yao and Chen(2006)], given a Concept Lattice $\mathcal{L}$, let $Ex(\mathcal{L})$ and $In(\mathcal{L})$ be the families of all the extents and all the intents of $\mathcal{L}$, respectively. Given a set of objects Q_o that is not the extent of any concept in $\mathcal{L}$, intuitively the upper approximation $\overline{apr}(Q_o)$ is the smallest set in $Ex(\mathcal{L})$ that contains Q_o, whereas the lower approximation $\underline{apr}(Q_o)$ is the largest set in $Ex(\mathcal{L})$ that is contained in Q_o. Formally we have:

$$\overline{apr}(Q_o) = \bigcap\{X \mid X \in Ex(\mathcal{L}), Q_o \subseteq X\}$$

$$\underline{apr}(Q_o) = \{X \mid X \in Ex(\mathcal{L}), X \subseteq Q_o, \forall X' \in Ex(\mathcal{L})(X \subseteq X' \Rightarrow X' \nsubseteq Q_o\}$$

Analogously, given the family of all the intents of $\mathcal{L}$, $In(\mathcal{L})$, consider a set of attributes Q_a. We can define the upper and lower approximations of Q_a, $\overline{apr}(Q_a)$ and $\underline{apr}(Q_a)$ respectively, as follows:

$$\overline{apr}(Q_a) = \bigcap\{X \mid X \in In(\mathcal{L}), Q_a \subseteq X\}$$

$$\underline{apr}(Q_a) = \{X \mid X \in In(\mathcal{L}), X \subseteq Q_a, \forall X' \in In(\mathcal{L})(X \subseteq X' \Rightarrow X' \nsubseteq Q_a\}$$

It is important to observe that, since $Ex(\mathcal{L})$ ($In(\mathcal{L})$) is closed under intersection, the smallest set containing Q_o (Q_a) is unique whereas this does not hold for the largest set contained in Q_o (Q_a). Therefore, lower approximations may not be unique.

In this section we have seen how a set of objects or a set of attributes can be approximated from above or from below by the extents or the intents, respectively, of concepts in FCA according to rough set theory. In the remaining of this paper we will focus on FFCA and, in particular, we will address the problem of Web search supported by rough set theory in FFCA.

4.1. Web Search based on Rough Sets and FFCA

In the following we assume we have a Fuzzy Concept Lattice and that Web queries are expressed by sets of attributes in the given formal context. Given a query, suppose there are no formal concepts having as intent the required set of attributes. Then, the goal is to find the formal concepts of the Fuzzy Concept Lattice whose intents "better approximate" the set of attributes specified by the query and, therefore, whose extents are closer to the expected answer. Furthermore, within the various approximations determined, the user can additionally select the preferred one on the basis of grades of membership of specific objects with specific attributes.

For instance, suppose we have the tourism ontology represented by the Fuzzy Concept Lattice of the *Sardinia Hotels* of Figure 2 and suppose the user is looking for an hotel on the sea where he/she can eat. This query can be represented by the following set of attributes:

$$Q_a = (Sea, Meal)$$

In the Fuzzy Concept Lattice of Figure 2 there are no concepts defined by the set of attributes Q_a. However we can look for the formal concepts whose intents better approximate it, i.e. the upper and lower approximations of Q_a. According to the definitions given above, the smallest intent in the Concept Lattice of Figure 2 containing Q_a is $(Theater, Meal, Sea)$, i.e., the concept whose intent is an upper approximation of Q_a is:

$$(\ ((H1, 0.7), (H3, 0.5)), \ (Theater, Meal, Sea) \ )$$

The grades of membership associated with the hotels $H1$ and $H3$, 0.7 and 0.5 respectively, specify "how much" these hotels are properly described by *both Sea* and *Meal*, but *also* by the attribute *Theater* which was not required by the user. For this reason, lower approximations of the query can be addressed. In this case, two lower approximations are identified in the Concept Lattice of Figure 2. They correspond to the following concepts whose intents are the largest sets contained in Q_a, i.e., the singletons $(Meal)$ and (Sea):

$$(\ ((H1, 1.0), (H3, 0.7), (H4, 1.0), (H6, 0.8)), \ (Sea) \ )$$
$$(\ ((H1, 1.0), (H2, 0.5), (H3, 0.5), (H5, 1.0)), \ (Meal) \ )$$

The user can therefore select as answer, on the basis of his/her needs, one of the concept extents associated with the above upper and lower approximations. In addition, within one of the above extents, he/she can choose the preferred objects also on the basis of his/her priorities according to the defined grades of membership. For instance, if Sea is the attribute with the highest priority, he/she can select the hotels $H1$ or $H4$, having both grades of membership with Sea equal to 1.0. Analogously, if $Meal$ is on the top of his/her preferences, the user can choose the hotels $H1$ or $H5$. In this case, by analyzing the lower approximations, it is reasonable to assume that the hotel $H1$ represents the closest answer to the user needs. Of course, this does not hold in general, and the user has to choose among several objects that do not perfectly match with the specified query. For instance consider the following query:

$$Q_a = (Tennis, SwPool, Meal).$$

Analogously to the previous example, in Figure 2 there are no concepts whose intents correspond

to the given set of attributes. Therefore, the upper approximation is addressed corresponding to the intent of the following concept:

(((H2, 0.5)), (*Tennis*,*SwPool*,*Meal*,*Cinema*))

and, furthermore, two lower approximations whose related formal concepts are:

(((H2, 0.6),(H4, 0.8),(H6, 0.8)), (*Tennis*,*SwPool*))
(((H1, 1.0),(H2, 0.5),(H3, 0.5),(H5, 1.0)), (*Meal*))

The user again can choose the answer on the basis of his/her needs by analyzing first the sets of attributes that better approximate the query. In particular he/she can select the hotel $H2$ having all the three required attributes and an additional one, *Cinema*, with grade of membership at least 0.5. Otherwise the user can choose the objects described by part of the required attributes with higher grades of membership. As opposed to the previous example, in this case there are no specific objects that fully satisfy the user needs. However, he/she can identify the favorite hotels by analyzing their grades of membership with preferred attributes (e.g. if *Tennis* and *SwPool* are both preferred, the hotels $H4$ and $H6$ could provide a satisfactory answer).

5. Related Work

Approximations in FCA have been investigated in several works in the literature. For instance in [Xu et al.(2008)] a generalized rough set model has been proposed aiming at providing a common framework for analyzing both FCA and Knowledge Spaces. In [Saquer and Deogun(2001)] two different approaches to concept approximation in FCA have been proposed. The first is based on rough set theory, the other on a similarity measure. The paper aims at comparing the different approaches and shows that the similarity-based algorithm is more efficient when the size of the Concept Lattice is not large. In [Yao and Chen(2006)], rough set approximations in FCA based on lattice-theoretic and set-theoretic operators are investigated. Analogously to the previous paper, the paper aims at comparing the two methods and shows that the set-theoretic approach, that is the one followed in our paper, provides a better understanding and solutions to existing problems in the study of data analysis.

In the perspective of developing the Semantic Web, the combination of rough set theory and FCA provides an interesting framework for the definition of hybrid similarity measures for ontology mapping, alignment, and integration [Zhao et al.(2007)]. With this regard, similarity measures for concepts in FCA have been defined in the literature following different approaches, see for instance [Formica(2006), Formica(2008)]. Furthermore, FCA can help ontology building by supporting the identification and the construction of concept hierarchies [Hwang et al.(2005)]. Several research efforts are emerging for representing and reasoning with vague and uncertainty information in the Semantic Web [Lukasiewicz and Straccia(2001), Lukasiewicz and Straccia(2009)] and for evaluating semantic similarity in FFCA [Formica(2010)]. Finally, in [Kim and Park(2001)] Web search has been performed by using dynamic keyword suggestion in the context of FCA (without employing rough sets).

Semantic Web search supported by rough set theory, independently of FCA, has been extensively analyzed in the literature. For instance, in [Ali and Mohd(2009)] an architecture of a Web search system has been presented that combines various evaluation techniques using a

rough set based rank aggregation technique, where ranking rules are learnt on the basis of the user feedback. In [Xu et al.(2001)], feedback information produced by the users are represented by rough sets and can influence the search results. In [Ngo and Nguyen(2005)] a method of snippet representation enrichment using Tolerance Rough Set model has been presented. In [Ziarko and Fei(2002)] an information retrieval methodology based on variable precision rough set model has been proposed where both queries and documents are represented as rough sets. In [Miao et al.(2006), Kumar De and Krishna(2004)] rough approximations have been used to cluster Web transactions from Web access logs, aiming at discovering Web page access patterns.

In [Doherty et al.(2003)], a formal framework for defining and automatically generating approximate concepts and ontologies from traditional crisp ontologies is presented, which is based on rough set theory. In [Jiang et al.(2009)], uncertain knowledge representation and reasoning in description logics are addressed, and a new rough description logic based on approximate concepts is proposed.

Finally *granular computing*, which is an emerging conceptual and computing paradigm, is employed in [Calegari and Ciucci(2010)] in order to have several granular perspectives for a specific ontology commitment, and in [Qiu et al.(2007)] in order to describe ontologies at different levels of granularity.

With respect to all the aforementioned works, in this proposal Semantic Web search is supported by Fuzzy FCA combined with rough set theory. In particular, in order to discover information in the Web the user can employ approximations of the queries and, in addition, thanks to the notion of fuzzy information in FCA, he/she can choose the preferred answer also on the basis of "grades of membership" that specify "how much" data are properly described by the required attributes. To our knowledge, Semantic Web search based on a combination of the above mentioned approaches has not been addressed in the literature.

6. Conclusion

In this paper rough set theory has been employed in combination with FFCA to perform Semantic Web search and discovery of information in the Web. In the case the required data are not modeled by any formal concept, the user can search and discovery the data that are closer to his/her preferences by following a twofold approach. He/she can select (i) super/subsets of the answer that are associated with lower/upper approximations of the query and, within the proposed answers, (ii) the data that are "better" described by the required attributes, on the basis of fuzzy values. As a future work, we intend to investigate how rough approximations can be employed to evaluate concept similarity in FFCA.

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