



**ISTITUTO DI ANALISI DEI SISTEMI ED INFORMATICA**  
"Antonio Ruberti"  
**CONSIGLIO NAZIONALE DELLE RICERCHE**

**F. Carravetta, G. Felici, P. Palumbo**

**FREQUENCY-BASED MODEL VALIDATION  
AND PARAMETER IDENTIFICATION OF A SEA-  
SURFACE VEHICLE**

**R. 09-12**

**Francesco Carravetta** – Istituto di Analisi dei Sistemi ed Informatica del CNR, Viale Manzoni 30, 00185 Roma, Italy. Email: [francesco.carravetta@iasi.cnr.it](mailto:francesco.carravetta@iasi.cnr.it).

**Giovanni Felici** – Istituto di Analisi dei Sistemi ed Informatica del CNR, Viale Manzoni 30, 00185 Roma, Italy. Email: [giovanni.felici@iasi.cnr.it](mailto:giovanni.felici@iasi.cnr.it).

**Pasquale Palumbo** – Istituto di Analisi dei Sistemi ed Informatica del CNR, Viale Manzoni 30, 00185 Roma, Italy. Email: [pasquale.palumbo@iasi.cnr.it](mailto:pasquale.palumbo@iasi.cnr.it).

ISSN: 1128–3378

Collana dei Rapporti  
dell'Istituto di Analisi dei Sistemi ed Informatica, CNR  
viale Manzoni 30, 00185 ROMA, Italy

tel. ++39-06-77161

fax ++39-06-7716461

email: [iasi@iasi.rm.cnr.it](mailto:iasi@iasi.rm.cnr.it)

URL: <http://www.iasi.rm.cnr.it>

## **Abstract**

This paper focuses on the modeling of the roll motion for a manned sea-surface vehicle. An extension of the Conolly model is proposed, where the uncertainties in the mechanical torques acting on the vehicle – torques coming out from wind and sea-wave action – are also included in a general state space representation. A frequency-based identification procedure is provided in order to identify both the ship and the disturbance parameters. The algorithm has been tested on real data and most significative results are proposed.

*Key words:* Stochastic Modelling; Validation; Identification Algorithms; Parameter Estimation; Frequency Signal Analysis



## 1. Introduction

In this paper we propose a stochastic linear model for the roll motion of a manned sea-surface vehicle, within a project financed by the Italian Ministry of Economic Development under the name of TMS (Trasmissioni Marine di Superficie). The final aim of the project is to design and effectively implement on board a prototype vehicle a model-based control algorithm in order to stabilize the roll motion. The vehicle is endowed with a couple of *flaps* that can be used as actuators by the controller in order to stabilize the roll-motion. Moreover the vehicle mounts a *trimmer*, i.e. a device able to vary the slope of the surface drive, which can be used as well in order to regulate the pitching of the watercraft. As far as the system's observations are concerned, our vehicle mounts real-time measurement devices for the pitch and roll angles.

A preliminary version of the model has been presented in [4], where a feedback is applied to give a linear-optimal performance with respect to a classical quadratic index (i.e. stochastic optimal control in the class of linear controllers [5]). However, in [4] neither model validation is performed, nor an identification algorithm is proposed to identify the model parameters.

The problem of stabilizing a watercraft against rolling has been widely studied in the literature. The roll-motion model here proposed is a linear second order, according to the Conolly theory (see [7]). Despite of its quite simple structure, it provides an exhaustive model for small roll displacements, and it has been widely adopted in the field of active fin control with the purpose of ship-roll reduction. We may cite, among the others, [9], where a pair of compensators are proposed: one designed by using classical frequency domain techniques, one obtained as an adaptive LQ compensator;  $H_\infty$  and robust adaptive  $H_\infty$  controllers are proposed in [20] and [11], respectively; in [1] a virtual instrumentation-based fin active control is obtained. A key-role in building the model is played by the external disturbance torque of the sea waves acting on the watercraft. Such an uncertain torque is mainly produced by the wind, but also affected by other atmosphere-conditions, ship moving, earthquake and gravitation of earth and moon. Many models are available in the literature (see [3], [19]). Generally, the sea waves are studied by means of wave energy spectrum (e.g. the Pierson-Moskowitz spectrum, see [18] or [17] for more details), according to the superposition of a large number of sinusoidal waves (see [19]).

In this work the sea wave/wind disturbances are modeled by a stochastic linear state space representation. As a matter of fact a not negligible set of parameters needs to be estimated on board, according to the weather/traveling conditions, because depending of the sea-strength and/or of the steady-state translational velocity of the vessel. For these reasons the parameter estimation algorithm should be pursued in a small time-interval (of the order of seconds) in order to be restarted on board when changes occur in weather/traveling conditions, and a detection-algorithm of these changes should also be provided in the control scheme. The present paper aims to yield an all embracing solution to the above issues by designing a suitable frequency-based identification algorithm, consisting of the optimization of an index weighting the gap between real and predicted spectral data.

The paper is self-contained and organized as follows. In §2 the watercraft's model is presented in details, together with some simulation results showing how different situations can be simulated by suitably tuning the sea wave/wind disturbance parameters. In §3 the identification strategy is described and in §4 the results of numerical simulations are presented, carried out on real data. Future work improvements towards the real time application on board a prototype vessel are shown in §5. Concluding remarks follow.

## 2. Stochastic linear model of the sea wave/wind disturbance

Denote with  $\alpha(t)$  the roll-angle of a sea-surface vehicle. According to the Conolly theory (see [7]), the linear roll motion equation in a single degree of freedom can be written as:

$$I_\alpha \ddot{\alpha}(t) + \bar{\beta} \dot{\alpha}(t) + \bar{k} \alpha(t) = \bar{k} \xi(t) + \bar{p} u(t), \quad (2.1)$$

where  $I_\alpha$  is the structural moment of inertia,  $\bar{\beta}$  is the damping coefficient relative to the surrounding water,  $\bar{k} \alpha(t)$  is the restoring moment,  $\xi(t)$  is the effective wave slope and  $\bar{p} u(t)$  denotes the control torque produced by the fins:  $\bar{p}$  is a coefficient depending of the translation-speed of the ship, and  $u(t)$  is the fin angular position, which is the control variable. Despite of its quite simple structure (indeed, it is a linear second order system) such a model is effectively exhausting in modeling the ship motion in many framework, see e.g. [19], [17], [3]. As a matter of fact, model (2.1) has been widely adopted in the field of active fin control, with the purpose of ship-roll reduction (e.g. [9], [20], [11], [1]).

By exploiting standard computations, eq.(2.1) may be written in the following first order ordinary-differential-equation (ODE) model:

$$\begin{aligned} \dot{\alpha}(t) &= \omega(t), \\ \dot{\omega}(t) &= -k \alpha(t) - \beta \omega(t) + k \xi(t) + p u(t). \end{aligned} \quad (2.2)$$

where  $\omega(t)$  is the roll angular speed and the coefficients  $k$ ,  $\beta$ ,  $p$  are given by:

$$k = \bar{k}/I_\alpha, \quad \beta = \bar{\beta}/I_\alpha, \quad p = \bar{p}/I_\alpha.$$

In the following the control input  $u(t)$  will be neglected, as this paper only focuses on the model-validating and parameter-estimation issues. Note that  $\xi(t)$  denotes the external and uncertain contribution to the whole torque applied to the ship. Such an external-torque is due to the sea/wind action and cannot be deterministically described, as we did for the other contributions. Indeed, it depends of the actual sea- and/or weather-conditions the ship is getting through. Differently from the standard frequency-based approaches [3], we assume here that  $\xi(t)$  admits the following linear state space stochastic representation, for a suitably fixed integer  $n$ :

$$\begin{aligned} dz(t) &= \Lambda z(t) dt + F dW(t), \quad z(t) \in R^n, \\ \xi(t) &= \Gamma z(t), \end{aligned} \quad (2.3)$$

where matrix  $\Lambda$  is Hurwitz (i.e. all eigenvalues have strictly negative real part) and  $W(t)$  is a standard Wiener process. Let us briefly comment on the above equations (2.3). If we guess, as it is reasonable, to the time-evolution of  $\xi(t)$  as to a somewhat 'irregular' periodic wave (reproducing indeed the shape of the water motion) then we can interpret matrix  $\Lambda$  as related to the wave 'frequency', responsible of the correlation between two distinct time instants. In fact, if  $t_1 = t$  and  $t_2 = t + \Delta t$ , then:

$$\begin{aligned} z(t + \Delta t) &= e^{\Lambda \Delta t} z(t) + V(t), \\ \xi(t + \Delta t) &= \Gamma e^{\Lambda \Delta t} z(t) + \Gamma V(t), \\ \xi(t) &= \Gamma z(t), \end{aligned} \quad (2.4)$$

with

$$V(t) = \int_t^{t+\Delta t} e^{\Lambda(t+\Delta t-\theta)} F dW_\theta,$$

a zero-mean Gaussian noise, independent of  $z(t)$ , whose covariance matrix is:

$$\text{Cov}(V(t)) = \int_0^{\Delta t} e^{\Lambda\theta} F F^T e^{\Lambda^T\theta} d\theta.$$

Then, the autocorrelation function of  $\xi(t)$  is given by:

$$\Psi_{\xi,\xi}(t, \Delta t) = \mathbb{E}[\xi(t + \Delta t)\xi(t)] = \Gamma e^{\Lambda\Delta t} \Psi_z(t) \Gamma^T,$$

with the covariance matrix  $\Psi_z(t) = \mathbb{E}[z(t)z^T(t)]$  obeying the differential equation:

$$\begin{aligned} \dot{\Psi}_z(t) &= \Lambda \Psi_z(t) + \Psi_z(t) \Lambda^T + F F^T, \\ \Psi_z(0) &= \Psi_{z,0}, \end{aligned}$$

from which:

$$\Psi_z(t) = e^{\Lambda t} \Psi_{z,0} e^{\Lambda^T t} + \int_0^t e^{\Lambda(t-\tau)} F F^T e^{\Lambda^T(t-\tau)} d\tau.$$

If we denote  $\lambda_1 = \max\{\Re[\lambda], \lambda \in \text{eig}(\Lambda)\} < 0$  (recall that  $\Lambda$  is Hurwitz), it is:

$$\begin{aligned} \|\Psi_z(t)\| &\leq e^{2\lambda_1 t} \|\Psi_{z,0}\| + \int_0^t e^{2\lambda_1(t-\tau)} \|F F^T\| d\tau \\ &= e^{2\lambda_1 t} \|\Psi_{z,0}\| + \frac{1 - e^{2\lambda_1 t}}{2|\lambda_1|} \|F F^T\|. \end{aligned}$$

By choosing  $\lambda_1$  more and more negative, the autocorrelation function of  $\xi(t)$  converges to zero:

$$\begin{aligned} \|\Psi_{\xi,\xi}(t, \Delta t)\| &\leq \|\Gamma\|^2 e^{\lambda_1 \Delta t} \|\Psi_z(t)\| \\ &\leq \|\Gamma\|^2 e^{\lambda_1 \Delta t} \left( e^{2\lambda_1 t} \|\Psi_{z,0}\| + \frac{1 - e^{2\lambda_1 t}}{2|\lambda_1|} \|F F^T\| \right) \mapsto 0 \end{aligned}$$

that is, the correlation between any two time instants vanishes as  $\lambda_1 \mapsto -\infty$ . As a matter of fact such a situation reflects a sea/wind disturbance close to a white noise sequence.

In the sequel, the following possible choice is adopted for the triple  $(\Lambda, F, \Gamma)$ :

$$\Lambda = \begin{bmatrix} \lambda & 1 & 0 & \cdots & 0 \\ 0 & \lambda & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 1 \\ 0 & \cdots & \cdots & 0 & \lambda \end{bmatrix}, \quad F = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \sigma \end{bmatrix}, \quad \Gamma = [1 \ 0 \ \cdots \ 0], \quad (2.5)$$

providing the following set of equations:

$$\begin{aligned} dz_1(t) &= \lambda z_1(t) dt + z_2(t) dt, \\ dz_2(t) &= \lambda z_2(t) dt + z_3(t) dt, \\ &\dots \quad \dots \quad \dots \quad \dots \quad \dots \\ dz_n(t) &= \lambda z_n(t) dt + \sigma dW(t), \\ \xi(t) &= [1 \ 0 \ \cdots \ 0] z(t), \end{aligned} \quad (2.6)$$

In this case, any occurrence of the function  $z$  shows a somewhat irregular shape but time-correlated: it does reproduce some sea-wave or wind shot action. Generally speaking, we point out that such a way to represent stochastic occurrences of functions has been used in the literature in very different settings. For instance in [10], in an image-processing framework, the occurrence of an image had been stochastically represented as being generated by the brightness-function derivatives up to a (high) fixed order where the derivative is assumed a (two-dimensional) white noise. Different settings of parameters  $n$ ,  $\lambda$ ,  $\sigma$ , within the structural choice of (2.5), provide different autocorrelation functions of the disturbance  $\xi(t)$ . Parameter  $\sigma$  in eq. (2.6) refers to the variance of the 'white noise', and is generally time-varying as far as the sea- and/or weather-conditions change. Nevertheless, since these changes in the overall weather-conditions reasonably are not frequent in the control time-horizon we are faced with, we assume here  $\sigma$  to be constant. We can interpret  $\sigma$  as related to the amplitude of the external-torque 'wave'. Thus we can think to  $\sigma$  as being a sea-strength related parameter: the bigger is the sea-strength, the larger is  $\sigma$ . The effectiveness of the choice proposed in (2.5) will be shown in the model-validation Section.

According to the Jordan block scheme (2.6), from (2.4):

$$\xi(t + \Delta t) = e^{\lambda \Delta t} \left( \xi(t) + \Delta t z_2(t) + \frac{(\Delta t)^{n-1}}{(n-1)!} z_n(t) \right) + V'(t),$$

with  $V'(t) = \Gamma V(t)$  a zero-mean Gaussian noise with:

$$\text{Cov}(V'(t)) = \int_0^{\Delta t} \Gamma e^{\Lambda \theta} F F^T e^{\Lambda^T \theta} \Gamma^T d\theta = \frac{\sigma^2}{(n-1)!} \int_0^{\Delta t} \theta^{2n-2} e^{2\lambda \theta} d\theta.$$

For  $n > 1$  the covariance of  $V'(t)$  becomes:

$$\text{Cov}(V'(t)) = \frac{\sigma^2 e^{2\lambda \Delta t}}{(n-1)!} \left[ \frac{(\Delta t)^{2n-2}}{2\lambda} + \sum_{k=1}^{2n-3} (-1)^k \frac{(2n-2) \cdots (2n-k-1)}{(2\lambda)^{k+1}} (\Delta t)^{2n-k-2} \right]$$

To test the effectiveness of the proposed scheme in modeling different disturbance frameworks, consider the simple (but quite exhaustive) case of  $n = 1$ . Then:

$$d\xi(t) = \lambda \xi(t) dt + \sigma dW(t),$$

from which:

$$\xi(t + \Delta t) = e^{\lambda \Delta t} \xi(t) + V(t),$$

with:

$$V(t) = \sigma \int_t^{t+\Delta t} e^{\lambda(t+\Delta t-\theta)} dW_\theta,$$

whose covariance matrix is:

$$\text{Cov}(V(t)) = \sigma^2 \int_0^{\Delta t} e^{2\lambda \theta} d\theta = \frac{\sigma^2}{2|\lambda|} (1 - e^{2\lambda \Delta t}).$$

Note that, for  $|\lambda| \mapsto 0$ , it comes:

$$\xi(t + \Delta t) \mapsto \xi(t) + V(t),$$

with  $\text{Cov}(V(t)) = \sigma^2 \Delta t$ , that is  $\xi(t)$  reduces to a pure Brownian motion. On the contrary, as already shown, for the more general scheme of (2.3) when  $\Re[\lambda] \mapsto -\infty$ ,  $\xi(t)$  reduces to a pure white noise.

Figs 2.1, 2.2, 2.3 report the disturbance  $\xi(t)$  evolution for the same values of  $n = 1$ ,  $\sigma = 0.01$ , on a time horizon of  $[0, 50]$  seconds. Parameter  $\lambda$  has been set equal to  $-0.01$  in fig.2.1, equal to  $-1$  in fig.2.2 and equal to  $-10$  in fig.2.3. Axis are in time (seconds) versus degrees.



Fig. 2.1 – Disturbance  $\xi(t)$  evolution:  $\lambda = -0.01$ .

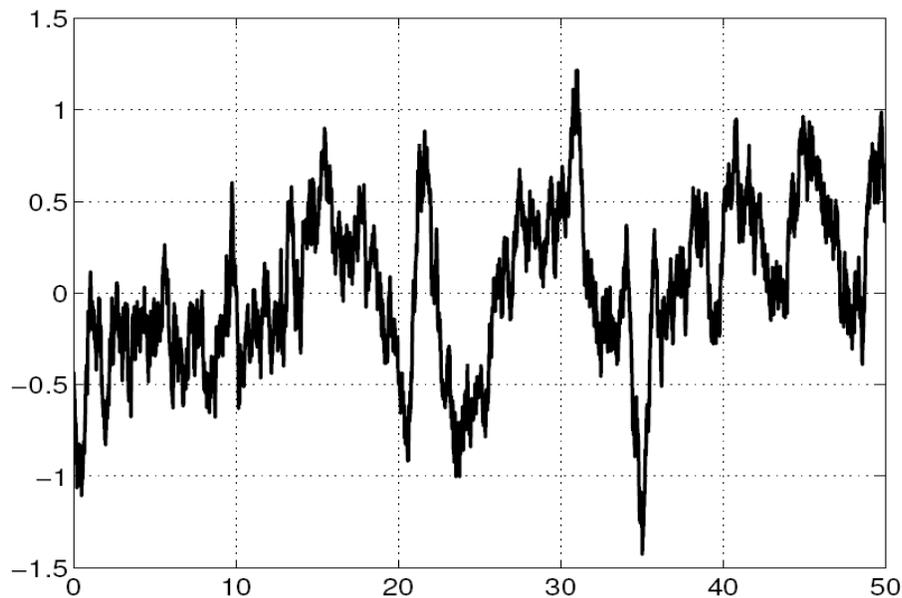


Fig. 2.2 – Disturbance  $\xi(t)$  evolution:  $\lambda = -1$ .

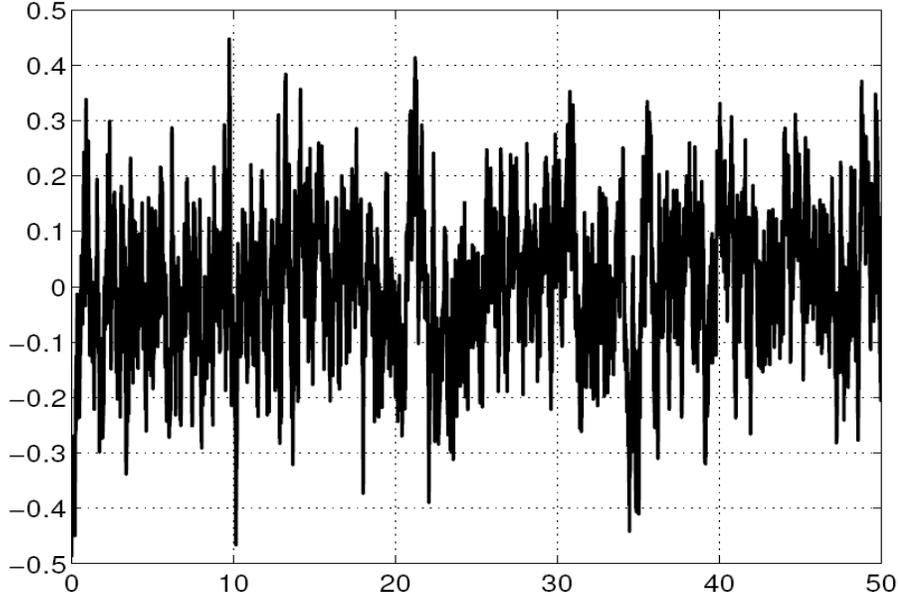


Fig. 2.3 – Disturbance  $\xi(t)$  evolution:  $\lambda = -10$ .

### 3. Model validation and identification procedure

In order to validate the effectiveness of the stochastic model of sea-wave uncertainties affecting the watercraft, an identification procedure is proposed, which aims to estimate the model parameters according to real data measurements. Such measurements are indeed available from the prototype vessel, which is being realized in the project mentioned so far (see Introduction). To this purpose a set of  $N$  samples of roll-angle's data is used, which comes out from a sensor actually placed on the watercraft: these measurements will be denoted by  $\mathcal{M}_N(\Delta t) = \{\alpha(k\Delta t), k = 1, 2, \dots, N\}$ ; where  $\Delta t$  is the sample time. The unknown parameter vector is given by the ship and the disturbance parameters:

$$\theta = [k \ \beta \ \lambda \ \sigma]^T \in \Theta,$$

where the parameter space  $\Theta$  is included in the positive orthant of  $\mathbb{R}^4$ .

As to the identification methods used here, it has to be stressed that resembling spectrum contents of the available data is more suited to our purposes than a direct comparison of their time-evolutions, as the latter ones are indeed just paths occurring from stochastic phenomena. Moreover, while hypothesizing (i) the ergodicity of the process at issue, and (ii) that the overall sea/weather conditions do not undergo changes in a suitably small time interval, the signal's spectrum can be assumed path-independent. Reasonably, both hypotheses (i), (ii) are satisfied in our case, so the identification procedure we are going to adopt will be based on simulated data's power-spectrum fitting the real data's one.

Spectral analysis is performed by using the Blackman and Tuckey smoothing window method, before taking the FFT on detrended data in order to estimate the power spectrum from real/simulated data [14]. Denote  $\mathcal{P}_{\mathcal{M}} = \{\rho(f_k), k = 1, \dots, N\}$  the power spectrum samples referred to real data and  $\mathcal{P}_{\theta} = \{\tilde{\rho}(f_k, \theta), k = 1, \dots, N\}$  the power spectrum samples referred to simulated data, according to a given quadruple of parameters  $\theta$  and to the same time feature of

the measured data (same sample time and horizon). The identification procedure is based on the minimization of the following index:

$$J_{\mathcal{P}\mathcal{M}}(\theta, m) = \sum_{k=1}^N |\rho(f_k) - \tilde{\rho}(f_k, \theta)|^m, \quad (3.1)$$

once the integer  $m$  has been chosen. Classical choices are  $m = 1$  or  $m = 2$ ; the limit case is given by  $m = +\infty$  (the sup-norm) which considers only the maximum error among the samples:

$$\tilde{J}_{\mathcal{P}\mathcal{M}}(\theta) = \max_{k=1, \dots, N} |\rho(f_k) - \tilde{\rho}(f_k, \theta)|. \quad (3.2)$$

As a matter of fact, such an index is related to a minimax identification approach. Also, a set of weights could be added to the index thus resembling this way just the frequencies we are mainly interested to.

#### 4. Simulations

The following simulations perform the identification procedure applied for a set of  $N = 2^7$  roll-angle time samples, with  $\Delta t = 0.4s$ , so that a time interval of about 50s is considered. Real data are reported in fig.4.1.

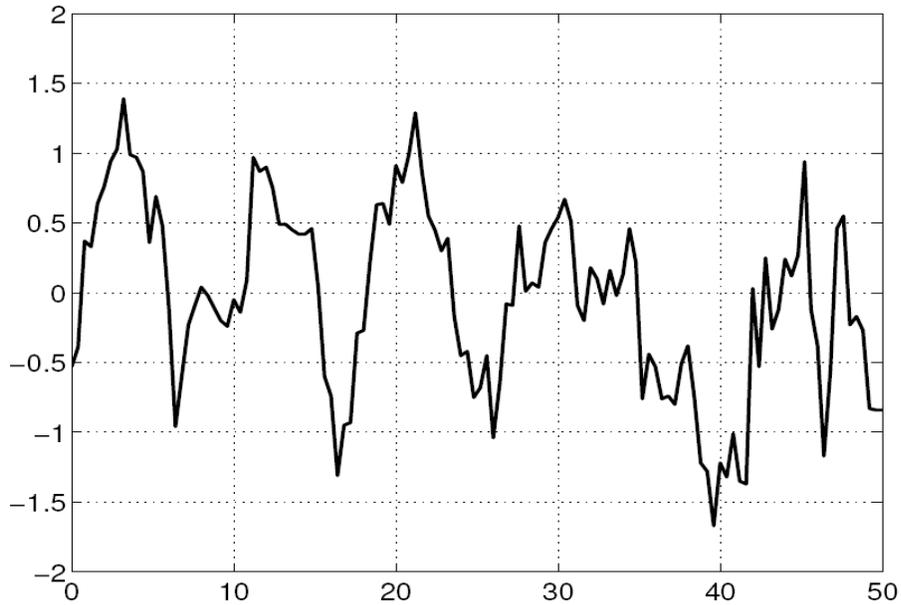


Fig. 4.1 – Real data evolution, time (seconds) vs degrees.

Numerical simulations are obtained according to the Euler-Maruyama algorithm (see [13]), with a discretization time  $dt = 0.01s$ , smaller than the sampling time  $\Delta t$ . Simulations here reported refer to the scheme of eq.s (2.6), with the order  $n$  fixed to 1. For  $m = 1$  the following estimates have been obtained:

$$k = 61.4 \quad \beta = 10.3 \quad \lambda = -0.42 \quad \sigma = 0.011.$$

Fig.4.2 pictures a comparison between the real and the simulated power spectrum, while fig.4.3 reports the simulated roll-angle time evolution. Note that fig.4.2 (and the others concerning spectra) shows periods (in seconds) vs power spectrum.

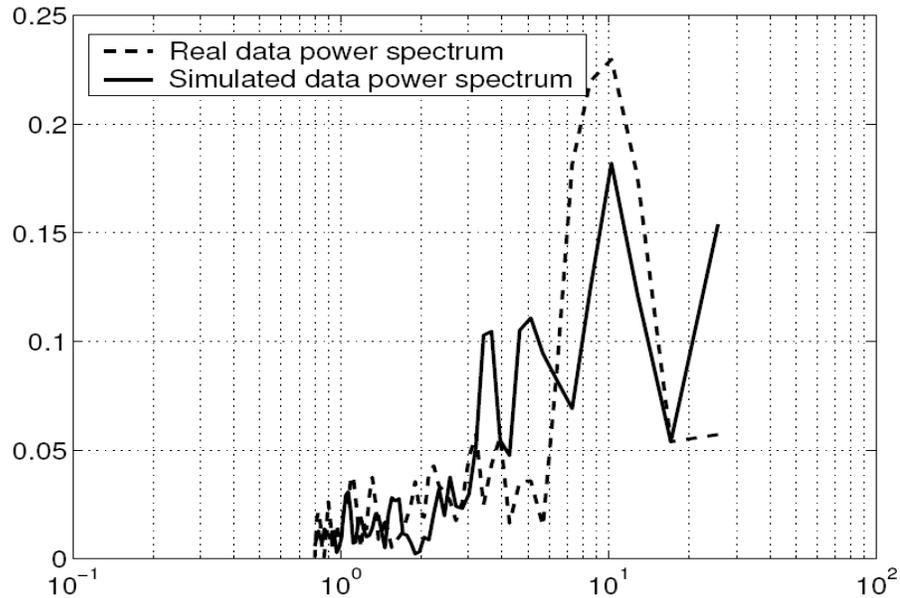


Fig. 4.2 – Real vs simulated Power Spectrum: eq.(3.1) with  $m = 1$ .

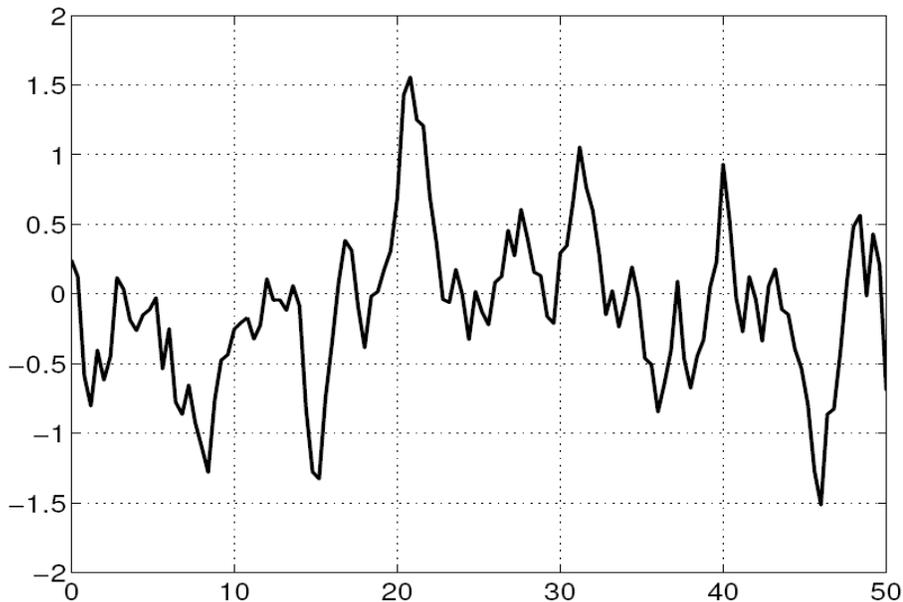


Fig. 4.3 – Simulated roll-angle evolution: eq.(3.1) with  $m = 1$ .

By changing  $m$  different estimates are obtained; for instance the case  $m = 2$  gives the following estimates:

$$k = 0.6 \quad \beta = 0.4 \quad \lambda = -1.43 \quad \sigma = 0.016,$$

according to which fig.4.4 displays the comparison between the real and the simulated power spectrum and fig.4.5 displays the new time domain simulated evolution.

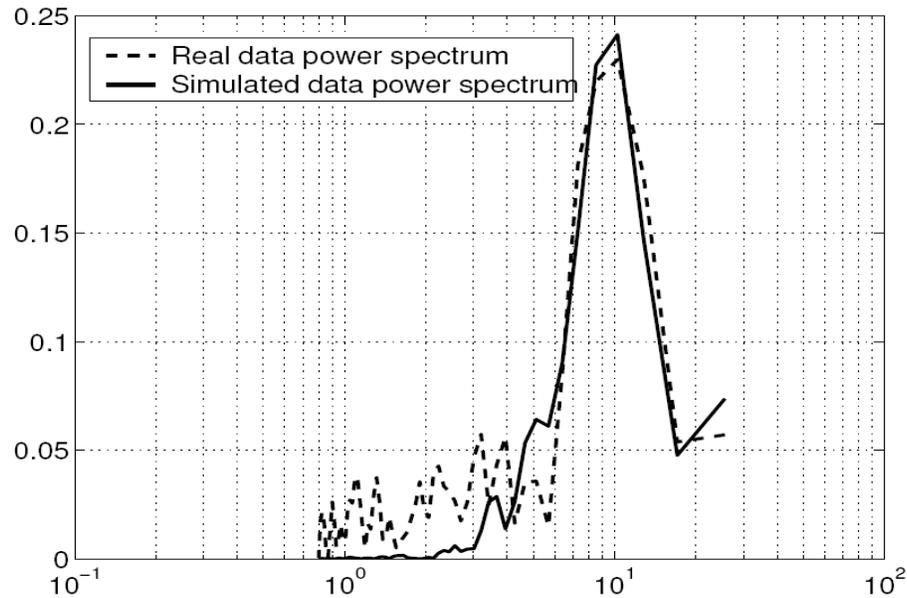


Fig. 4.4 – Real vs simulated Power Spectrum: eq.(3.1) with  $m = 2$ .

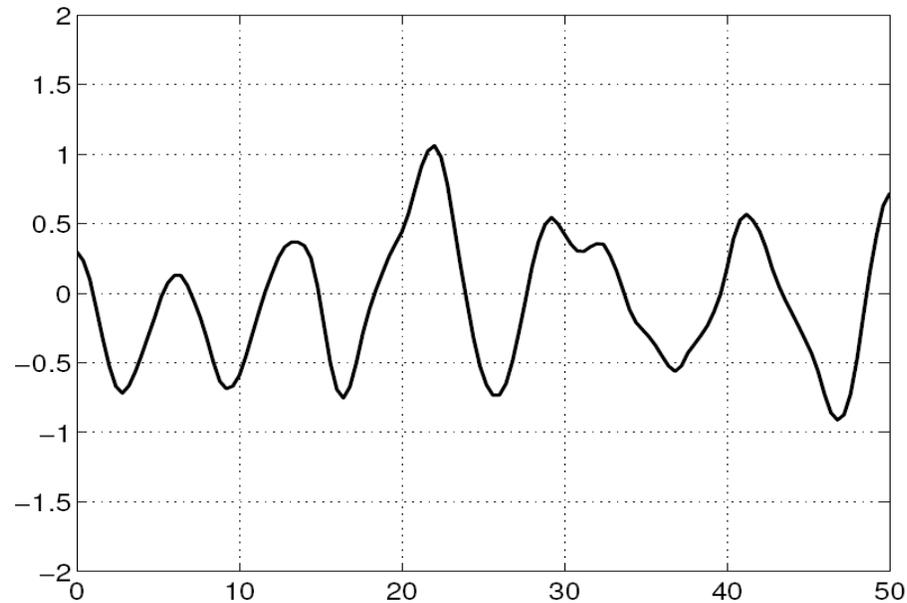


Fig. 4.5 – Simulated roll-angle evolution: eq.(3.1),  $m = 2$ .

Let us comment on these results. By applying the optimization algorithm, increasing  $m$  results in a smoother time evolution. Indeed, from fig.4.4, it appears that when  $m = 2$  is chosen, most of the higher frequencies are neglected in favor of the dominant ones. On the other hand, according to a qualitative inspection, when looking at the time evolutions of figs.4.1 and 4.5 (real and simulated data, respectively) we count roughly the same number of major peaks, besides the superimposed ripples occurring only in the real data time course (fig.4.1), which are due to the high-frequency contributions. On the contrary, high-frequency ripples occur in the time evolution related to the choice of  $m = 1$  (fig.4.3), but low-frequency contributions are not

so clearly apparent.

It has to be stressed that it is expected just the simulated power spectra resemble the real ones: in fact, at this stage, we are not making a prediction of the roll angle, but just estimating the vector parameter  $\theta$ , whose entries are related to the mechanical features of the ship ( $\beta$  and  $k$ ) and to the statistical features of the sea/wind disturbances ( $\lambda$  and  $\sigma$ ).

Finally, the case of index (3.2) has also been considered, whose optimal parameters result to be quite close to the ones of index  $m = 2$ :

$$k = 0.5 \quad \beta = 0.4 \quad \lambda = -1.49 \quad \sigma = 0.02.$$

In summary, it appears that index (3.1) with  $m = 1$  is the only capable to provide the complete spectrum variability of the given data, whence by increasing  $m$  just the fundamental is caught with good precision. In order to distinguish which approach should be preferred, we have tested our algorithm with the aim of data prediction. To this purpose we have added a zero-mean Gaussian noise to our data (with standard deviation  $\sigma_{out} = 0.002$ ), and then we have applied the Kalman-Bucy Filter (KBF) with a ship-model whose parameters are the one estimated for  $m = 1$  and  $m = 2$ . In this case, it comes out from figs.4.6 and 4.7 that the first choice provides better estimates.

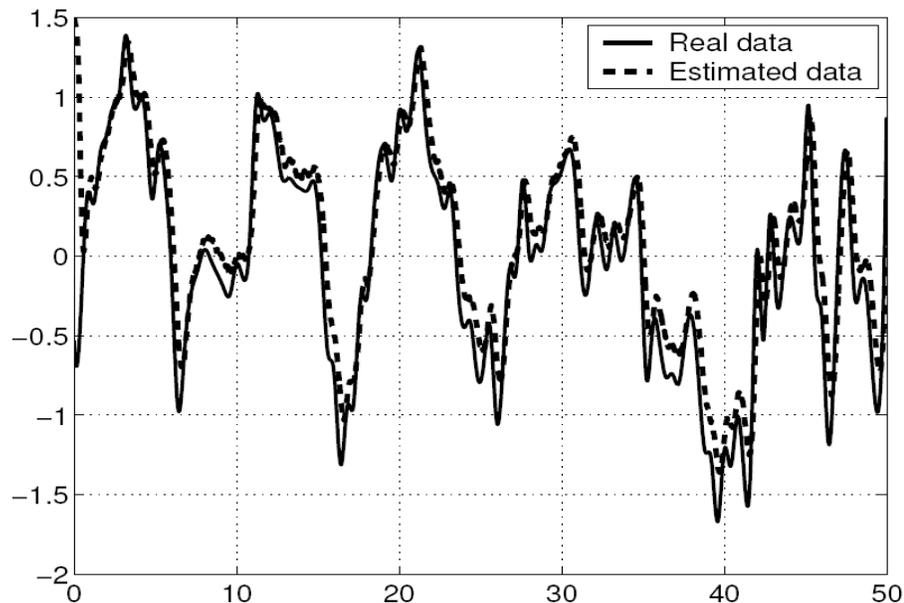


Fig. 4.6 – KBF: the case of (3.1) with  $m = 1$ .

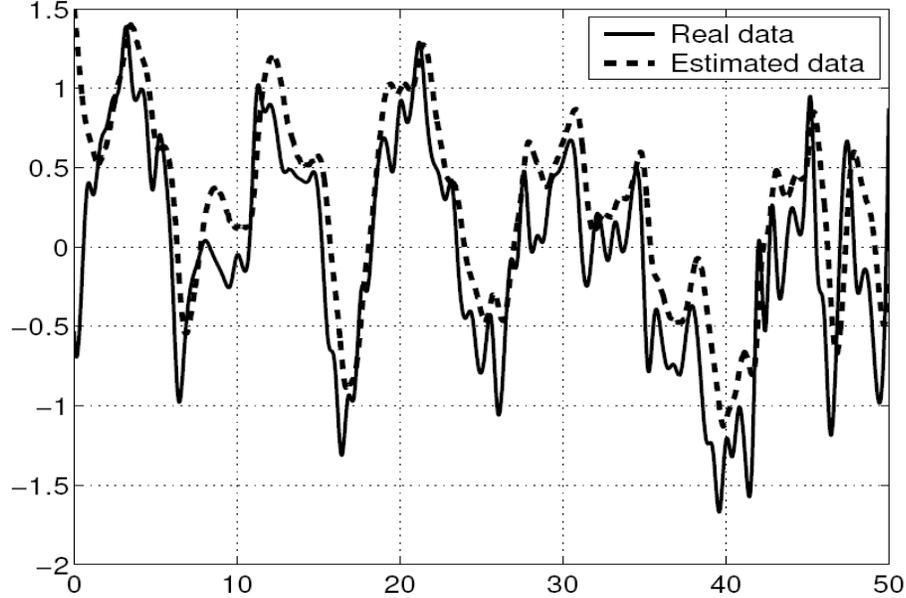


Fig. 4.7 – KBF: the case of (3.1) with  $m = 2$ .

## 5. Future works: towards the real time application

Despite the solutions found with simulation are of a good quality, we are facing the problem of applying the described method in real time on a vessel with limited computation availability, in order to be applied each time the weather/traveling conditions change, according to a supervisor device. For this reason, we have started to experiment a global optimization method to find the best value for the model parameters with a computational time that is reasonable with the application, i.e., very close to real time (one may envision to invest in such phase at most few seconds while navigating).

To this aim, assume the variable  $\theta$  is constrained to remain in a region  $\Theta \subseteq \mathbb{R}^4$  defined by upper and lower bounds as follows

$$\Theta = \{\theta \in \mathbb{R}^4 : l_i \leq x_i \leq u_i, \quad i = 1, \dots, 4\} \quad (5.1)$$

and  $|l_i|, |u_i| < +\infty$ , that is to say that the bounds on the variables are proper. Hence the problem that we want to solve to global optimality is (3.1) (or (3.2)), with  $\Theta$  given by (5.1). Such a problem has some distinguishing features which we can take into proper account. First of all, the objective function is the result of complex and possibly noisy observations and simulations. For this reason, it would be better not to involve first order derivatives (or their approximations) in the optimization algorithm. Second, many local minimum points may be present besides the global ones. Finally, the number of variables, that is 4, is relatively small.

On the basis of the above-mentioned problem characteristics we chose to tackle such a minimization problem by the well-known derivative-free global optimization method **DiRect** [16, 15]. Since its introduction [16] **DiRect** has received great attention by the optimization community, and has been widely used for solving optimal design problems [8,2,12,6].

**DiRect**, which stands for *Divide Rectangles* is based on the idea of partitioning the feasible set  $\Theta$  into a growing number of hyperintervals

$$\Theta^j = \{\theta \in \mathbb{R}^4 : l^j \leq \theta \leq u^j\}, \quad j \in I_k, \quad (5.2)$$

where  $k$  is the iteration counter,  $I_k = \{1, \dots, r_k\}$  and  $l^j, u^j$  are the vectors in  $\mathbb{R}^4$  of the lower, upper bounds determining the hyperinterval  $\Theta^j$  (the inequalities in (5.2) have to be considered element-wise). At each iteration, some hyperintervals, namely the more *promising* ones, are selected, further partitioned, and the solutions there contained are considered.

Further details on the algorithm can be found in the related literature and go beyond the purpose of this paper; suffice it to say that, in the limit, when the number of iterations grows to infinity, the method produces a dense set of points and is therefore possible to prove convergence to a global minimum point of the objective function. These strong theoretical properties make `DiRect` the method of choice for problems of moderate size like the one at hand.

The initial experiments conducted with `DiRect` already exhibit a large reduction of the computing time needed to estimate the parameters and are therefore encouraging for further application of the method in the control system. The results on a larger test set and real application on an experimental vessel will be treated in future communications.

## 6. Conclusions

A linear stochastic state space representation of the sea wave/wind disturbance has been proposed, coupled to the linear roll motion equations based on the Conolly theory. The model has been validated according to real data measurements, and an identification procedure is also given, according to which it has been shown how the proposed model allows to capture the actual wave behavior. It has to be stressed that according to [4], where the optimal control law was implemented in a simulation framework and tentative values had been chosen for the parameters, the model-based approach resulted to be effective even assuming a not complete knowledge upon the parameters. The algorithm is currently being implemented on the devices which will be mounted on the real watercraft, while a control-system test is being planned.

## Acknowledgements

The authors wish to thank Dr. Giampaolo Liuzzi and Dr. Enrico Mastrostefano for their precious work and effective suggestions in implementing the proposed identification scheme.

## References

- [1] V. Anantha Subramanian, G. Asokumar, and V. Jagadeesh Kumar, "Active fin control for yacht using virtual instrumentation," *Ocean Engineering*, Vol.34, pp.390–402, 2007.
- [2] M.C. Bartholomew-Biggs, S.C. Parkhurst, S.P. Wilson, "Using DIRECT to solve an aircraft routing problem," *Computational Optim. and Appl.*, Vol.21, pp.311–323, 2002.
- [3] R. Bhattacharyya, *Dynamics of Marine Vehicles*, New York: Wiley, 1978.
- [4] F. Carravetta, G. Felici and P. Palumbo, "Regulation of a Manned Sea-Surface Vehicle via Stochastic Optimal Control," *IFAC World Conference on Automatic Control*, Seoul, South Korea, July 2008.
- [5] F. Carravetta, and G. Mavelli, "Suboptimal Stochastic Linear Feedback Control of Linear Systems with State-dependent Noise: the Incomplete Information Case," *Automatica*, Vol.43, pp.751–757, 2007.
- [6] R.G. Carter, J.M. Gablonsky, A. Patrick, C.T. Kelley, O.J. Eslinger, "Algorithms for noisy problems in gas transmission pipeline optimization," *Optim. and Engineering*, Vol.2(2), pp.139–157, 2001.
- [7] J.E. Conolly, "Rolling and its stabilization by active fins," *Transactions of the Royal Institute of Naval Architects*, Vol.111, pp.21–48, 1968.
- [8] S.E. Cox, R.T. Haftka, C.A. Baker, B. Grossman, W.H. Mason, L.T. Watson, "A comparison of global optimization methods for the design of high-speed civil transport," *J. Global Optim.*, Vol.21, pp.415–433, 2001.
- [9] L. Fortuna and G. Muscato, "A roll stabilization system for a monohull ship: modeling, identification and adaptive control," *IEEE Transactions on Contr. Syst. Tech.*, Vol.4, pp.18–28, 1996.
- [10] A. Germani and L. Jetto, "Image modeling and restoration: A new approach," *Cir. Syst. Signal Proc.*, Vol.7, pp.427–457, 1988.
- [11] Z. Haipeng, Y. Xuliang, C. Fang, and J. Hongzhang, "Research on lift feedback fin stabilizer and its nonlinear controller," *Proc. of SICE annual conference in Fukui*, Fukui, Japan, pp.1844–1849, 2003.
- [12] J. He, L.T. Watson, N. Ramakrishnan, C.A. Shaffer, A. Verstak, J. Jiang, K. Bae, W.H. Tranter, "Dynamic data structures for a Direct search algorithm," *Computational Optim. and Appl.*, Vol.23, pp.5–25, 2002.
- [13] D.J. Higham, "An algorithmic introduction to numerical simulation of stochastic differential equations," *SIAM Review*, Vol.43, pp. 525–546, 2001.
- [14] G.M. Jenkins and D.G. Watts, *Spectral analysis and its applications*, Holden-Day, San Francisco, 1968.
- [15] D.R. Jones, "The DIRECT global optimization algorithm", *C. Floudas, P. Pardalos (edit.): "Encyclopedia of Optimization"*, Kluwer Academic Publishers, Dordrecht, pp.431-440, 2001.
- [16] D.R. Jones, C.D. Perttunen, B.E. Stuckman, "Lipschitzian optimization without the lipschitz constant," *J. of Optim. Theory and Appl.*, Vol.79(1), pp.157–181, 1993.
- [17] E.V. Lewis and J.P. Comstock, *Principles of Naval Architecture*, New York: Society of Naval Architects and Marine Engineers, 1967.
- [18] W.J. Pierson and L. Moskowitz, "A proposed spectral form for fully developed wind seas based on the similarity theory of S.A. Kitaigorodskii," *Trans. American Geophysical Union*, Vol.69, 1964.
- [19] N.G. Price and R.E.D. Bishop, *Probabilistic Theory of Ship Dynamics*, London: Chapman and Hall, 1974.

- [20] L. Sheng, S. Jingchuan, and C. Shengzhong, "Ship's fin stabilizer  $H_\infty$  control under sea wave disturbance," *Proc. of the 1999 IEEE Canadian Conference on Electrical and Computer Engineering*, pp.891–895, 1999.