



ISTITUTO DI ANALISI DEI SISTEMI ED INFORMATICA
CONSIGLIO NAZIONALE DELLE RICERCHE

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**SIMILARITY REASONING FOR FUZZY
CONCEPT LATTICES**

R. 09-02 2009

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ISSN: 1128–3378

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Abstract

Fuzzy Formal Concept Analysis (FFCA) is a generalization of *Formal Concept Analysis* (FCA) for modeling uncertainty information, which is revealing interesting in supporting critical activities for the development of the Semantic Web, such as *Similarity Reasoning*. In this paper the definition of a similarity measure for FFCA concepts is proposed as a combination of the similarity of concept extents (fuzzy sets) and the similarity of the information contents of concept intents.

1. Introduction

Similarity Reasoning, i.e. the identification of syntactically different concepts that are semantically close, is becoming fundamental for the retrieval and integration of information within the Semantic Web. Similarity Reasoning becomes more important in the presence of vague information, when some data are more relevant than others, or when a feature does not fully describe (or is not totally appropriate to) the resource the user is looking for. This type of problems can be tackled with fuzzy information [12]. *Fuzzy Formal Concept Analysis* (FFCA) is a generalization of *Formal Concept Analysis* (FCA) [10] for modeling uncertainty information [2]. It is revealing interesting in supporting critical activities for the development of the Semantic Web, such as ontology mapping, integration, alignment, and Similarity Reasoning [1, 9].

In this article a measure for evaluating the similarity of FFCA concepts is proposed, originating from a work for (non-fuzzy) Concept Lattices [5]. With respect to other papers defined in the literature, this proposal allows FFCA concept similarity to be evaluated as a combination of the similarity of concept extents (fuzzy sets) and concept intents. In particular, concept intents are compared according to the *information content* approach [8, 6], which has a higher correlation with human judgement than traditional approaches.

Concept Lattices

In FCA a *context* is a triple (O, A, R) , where O is a set of *objects*, A is a set of *attributes*, and R is a binary relation between O and A . For instance, consider a context named *Sardinia Hotels*, suppose that the set O is defined by the following six objects representing six different hotels:

$$O = \{H1, H2, H3, H4, H5, H6\},$$

and the set A is defined by three possible attributes of these objects:

$$A = \{SwPool, Sea, Meal\}$$

where *SwPool* stands for swimming pool. Furthermore, suppose the hotels are related to the above attributes according to the binary relation R defined by the following table:

	SwPool	Sea	Meal
H1		x	x
H2	x		x
H3		x	x
H4	x	x	
H5		x	x
H6	x	x	

Table 1: The *Sardinia Hotels* context in (non-fuzzy) FCA

Table 1 establishes that, for instance, the hotel *H2* has two attributes, namely *SwPool* and *Meal* and, viceversa, both attributes *SwPool* and *Meal* apply to the object *H2*. Given a context, in FCA a concept is a pair (E, I) , where the former element, referred to as concept *extent*, is a set consisting of precisely those objects having all the attributes from the latter and, conversely, the latter, referred to as concept *intent*, is a set containing precisely those attributes that apply to all the objects from the former.

4.

For instance, a concept of the *Sardinia Hotels* context is:

$$((H1, H3, H5), (Sea, Meal))$$

since all the $H1, H3, H5$ objects have both the attributes Sea and $Meal$, and viceversa, both these attributes only apply to the objects $H1, H3, H5$.

Given two concepts of a context, (E_1, I_1) , (E_2, I_2) , it is possible to establish an *inheritance relation* (\leq) between them according to the following condition:

$$(E_1, I_1) \leq (E_2, I_2) \text{ iff } E_1 \subseteq E_2 \text{ (iff } I_2 \subseteq I_1).$$

In particular, (E_1, I_1) is called *subconcept* of (E_2, I_2) and (E_2, I_2) is called *superconcept* of (E_1, I_1) .

Given a context (O, A, R) , consider the set of all the concepts of this context, indicated as $\mathcal{L}(O, A, R)$. Then:

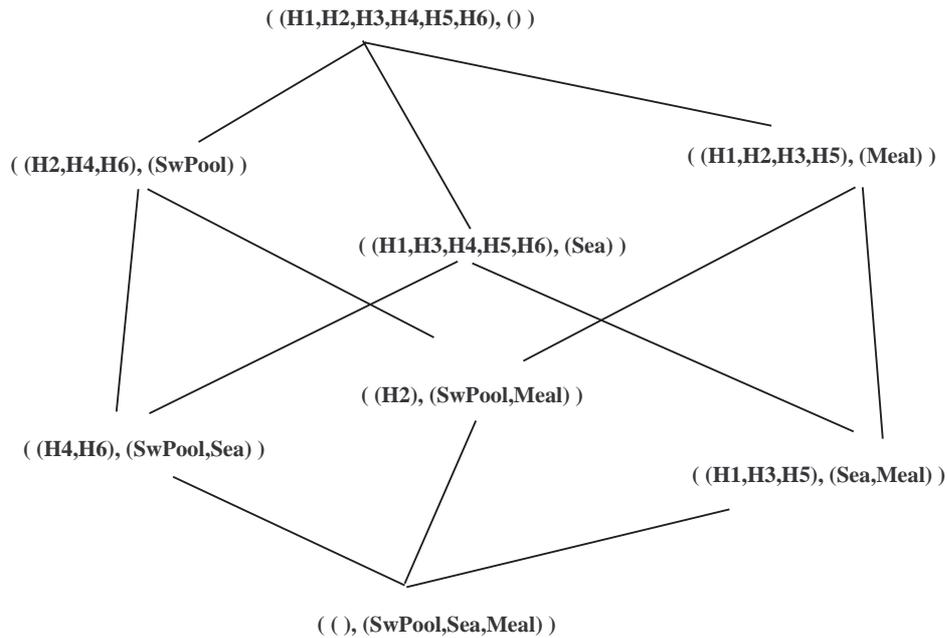
$$(\mathcal{L}(O, A, R), \leq)$$

is a complete lattice called *Concept Lattice* (also referred to as *Galois Graph*), i.e., for each subset of concepts, the greatest common subconcept and the least common superconcept exist [10]. For instance, the Concept Lattice that can be constructed from the context of Table 1 is shown in Figure 1. Note that nodes are labeled with the concepts of the context, and arcs are established among the nodes whose associated concepts are in \leq relation. The Concept Lattice also has two special nodes, the maximum and minimum nodes, grouping all the objects and the attributes of the context, respectively. In Figure 1, the least common superconcept of, for instance, the concepts $((H4, H6), (SwPool, Sea))$ and $((H1, H3, H5), (Sea, Meal))$ is the concept $((H1, H3, H4, H5, H6), (Sea))$. Such a concept has the set of objects and the set of attributes defined by the union and the intersection of the sets of objects and the sets of attributes of the given concepts, respectively. Whereas the greatest common subconcept of the concepts $((H2, H4, H6), (SwPool))$ and $((H1, H2, H3, H5), (Meal))$ is the concept $((H2), (SwPool, Meal))$, having as set of objects and set of attributes the intersection and the union of the sets of objects and the sets of attributes of the concepts, respectively.

Unfortunately, modeling a domain of interest with traditional FCA (i.e., with non-fuzzy sets) can be inaccurate when the attributes do not describe the objects in a uniform way or, in other words, a given attribute applies to different objects in different ways. For instance, in our example, consider the attribute Sea . One should be able to distinguish the hotels located just on the sea, from that having a walking distance seaside (reachable in, let us say, ten or twenty minutes). Analogously, regarding the attribute $Meal$, we would like to be aware about the hotels providing both lunch and dinner, rather than half-board. Without the introduction of fuzzy information, we have no way to specify how appropriate is a feature, or an attribute, to a given object, therefore describing all the objects in a uniform way.

2. Fuzzy Concept Lattices

A *fuzzy set* A in a space of points X is characterized by a *membership function* $\mu_A(x)$ which associates with each point x in X a real number in the interval $[0,1]$ representing the *grade of*

Figure 1: Concept Lattice of the *Sardinia Hotels* context

membership of x in A . Note that for an ordinary set, the membership function can take only the values 1 and 0, depending on x does or does not belong to A , respectively. Just to provide an example, assume X is a set of people, a fuzzy set *Young* is defined by associating with each person in X a real number in $[0,1]$ establishing the degree of youth of the person, such that the nearer this value to unity, the higher the grade of membership of a person in the set *Young*. The notion of a *fuzzy relation* can be obtained by generalizing the notion of a fuzzy set as follows. A fuzzy relation R in $X \times Y$, is a fuzzy set in the product space $X \times Y$.

For instance, consider the *Sardinia Hotels* context with fuzzy information. This context, referred to as *fuzzy context*, is specified by the fuzzy relation given in Table 2. In particular, crosses in Table 1 have been replaced by grades of membership, from 0 to 1, each allowing us to quantify "how much" an object has, or is described by, an attribute and viceversa an attribute applies to an object.

	SwPool	Sea	Meal
H1		1.0	1.0
H2	1.0		0.5
H3		0.7	0.5
H4	1.0	1.0	
H5		0.3	1.0
H6	1.0	0.8	

Table 2: The *Sardinia Hotels* context in FFCA

For instance, consider the hotel $H2$ in Table 2. It has the attribute *SwPool* with grade of membership 1.0, which means that such attribute fully applies to the hotel $H2$ (and viceversa the hotel $H2$ can be properly described by the attribute *SwPool*). Instead, the object $H2$ has

the attribute *Meal* with a membership value 0.5, which means that such an attribute partially applies to this hotel (for instance it could provide meals just for lunch or dinner). Analogously, in the case of *H3*, the value 0.7 in correspondence with the attribute *Sea* means that this feature better describes the hotels *H1*, *H4* or *H6* than *H3*, but it is more appropriate to *H3* than *H5* (having *H5* a low grade of membership with *Sea*, i.e., 0.3). However, in order to address only objects related to attributes with relevant grades of membership, a threshold is fixed such that the pairs with membership values less than the threshold are ignored. For instance, consider our running example and assume that a threshold is fixed equal to 0.5. A fuzzy concept of the *Sardinia Hotels* fuzzy context is, for instance, the pair:

$$((H1,1.0),(H3,0.5)), (Sea,Meal)$$

In fact the objects *H1*,*H3* share both the attributes *Sea* and *Meal* and, viceversa, both attributes *Sea* and *Meal* only apply to the objects *H1* and *H3*, with membership values which are not less than the threshold. With respect to the previous example without fuzziness, the object *H5* is not present due to the threshold. It is important to note that in a concept, in the case the attributes apply to an object with different grades of membership, the minimum among them is selected. For instance, the object *H3* has the attributes *Sea* and *Meal* with different grades of membership, that are 0.7 and 0.5, respectively. Therefore, in the concept above, the minimum value between them has been selected because it represents the highest common grade of membership that allows *H3* to be described by both the attributes *Sea* and *Meal*.

Fuzzy Concept Lattices can be defined similar to Concept Lattices, on the basis of fuzzy set theory [12]. Informally, given two fuzzy sets *A*,*B*, the fuzzy set intersection and fuzzy set union are defined by fuzzy sets where the membership functions are the minimum and maximum of the membership functions of *A* and *B*, respectively. For instance, the Fuzzy Concept Lattice that can be constructed from the context of Table 2 is shown in Figure 2. Analogously to Concept Lattices, nodes are labeled with the fuzzy concepts of the context, and arcs are established among the nodes that are in inheritance relation. The Fuzzy Concept Lattice also has two special nodes, the maximum and minimum nodes, grouping all the objects and the attributes of the context, respectively. In particular, the membership values associated with the objects of the maximum node are all equal to one. Also in Fuzzy Concept Lattices, for any subset of concepts, the greatest common subconcept and the least common superconcept are always defined. For instance, consider the concepts:

$$((H2, 1.0),(H4, 1.0),(H6, 1.0)), (SwPool)$$

$$((H1, 1.0),(H3, 0.7),(H4, 1.0),(H6, 0.8)), (Sea)$$

The greatest common subconcept is:

$$((H4, 1.0),(H6, 0.8)), (SwPool, Sea)$$

where the fuzzy set intersection of the objects and the set union of the attributes has been performed. As mentioned above, fuzzy set intersection requires that the minimum among different grades of membership associated with the same object is selected (in the case of *H6*, 0.8).

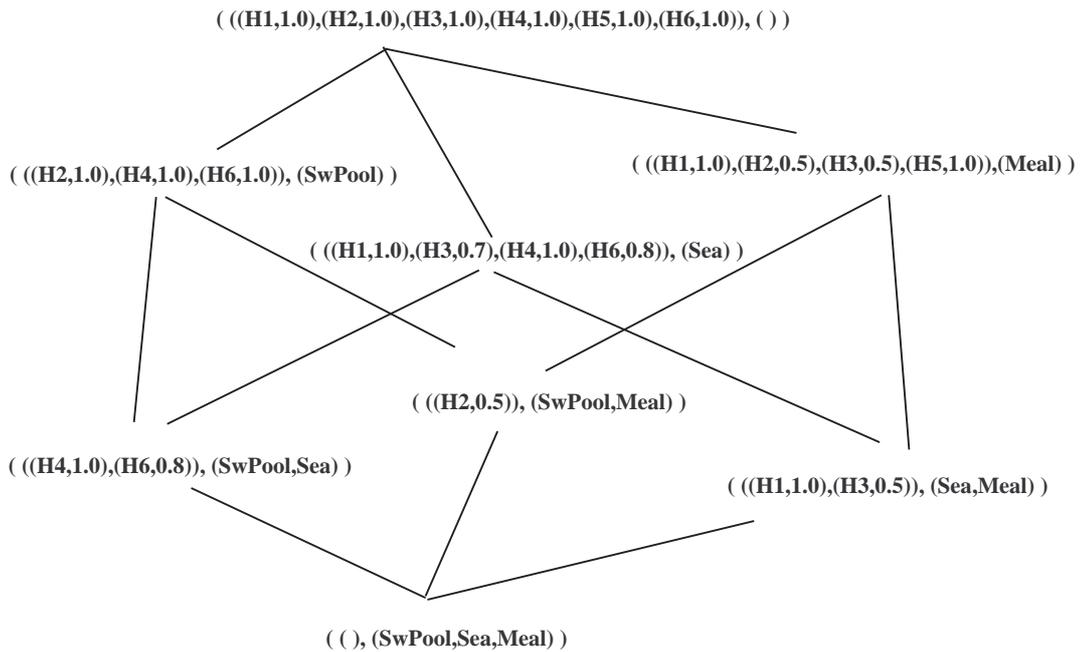


Figure 2: Fuzzy Concept Lattice of the *Sardinia Hotels* context

3. Information Content Similarity

In this paper, the notion of *information content similarity* will be recalled [8, 6], that allows similarity of concept intents (attributes) to be computed.

Let us consider a *lexical database for the English language* as, for instance, *WordNet* [11]. Besides the English nouns, *WordNet* contains verbs, adjectives and adverbs, each associated with the related natural language definition and *frequency*. Frequencies are estimated using noun frequencies from large text corpora, as for instance the *Brown Corpus of American English*. Nouns are organized according to the *ISA* and *PartOf* relationships, and for each noun, a set of synonyms is given (*SynSet*). The *probability* of a concept noun n , $p(n)$, is defined as:

$$p(n) = \frac{freq(n)}{M}$$

where $freq(n)$ is the frequency of n from a text corpus, and M is the total number of observed instances of nouns in the corpus. In this paper probabilities have been assigned according to the *SemCor* project [3], which labels subsections of the *Brown Corpus* to senses in the *WordNet* lexicon.

In this paper a *weighted ISA hierarchy* is a fragment of *WordNet* ISA hierarchy, where each noun n is associated with the probability $p(n)$.

For instance, below the definitions of *Water*, *Lake*, *Stream*, *Beach*, and *Sea* are given according to *WordNet*, and their frequencies (the number in parenthesis):

- (219) *Water* – the part of the earth’s surface covered with water (such as a river or lake or ocean);
- (3) *Lake* – a body of (usually fresh) water surrounded by land;
- (20) *Stream* – a natural body of running water flowing on or under the earth;
- (14) *Beach* – an area of sand sloping down to the water of a sea or lake;

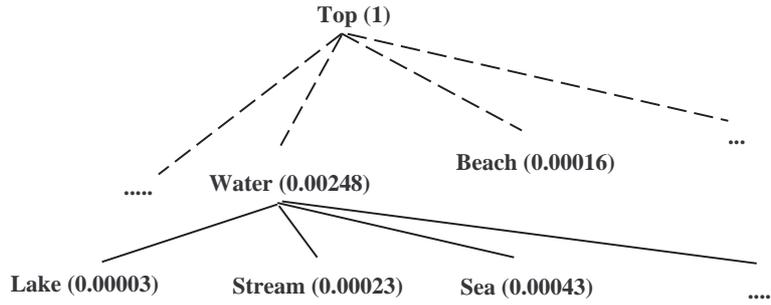


Figure 3: A weighted ISA hierarchy derived from *WordNet*

(38) *Sea* – a division of an ocean or a large body of salt water partially enclosed by land;

....

A weighted ISA hierarchy derived from *WordNet* is shown in Figure 2 (note that dotted lines stand for undirected ISA links).

Just to show a small set of sets of synonyms of *WordNet* that are relevant to our example, we have:

$$\begin{aligned} \text{SynSet} = \{ & \dots, \\ & \{ \textit{Stream}, \textit{Watercourse} \}, \\ & \{ \textit{Water}, \textit{Body_of_water} \}, \\ & \dots \} \end{aligned}$$

Once probabilities have been associated with nouns, the starting assumption of the approach is that the *information content* of a noun n is defined as $-\log p(n)$, that is, as the probability of a concept noun increases, the informativeness decreases, therefore the more abstract a concept noun, the lower its information content.

For instance, consider the weighted ISA hierarchy of Figure 3. *Water* is a concept noun more abstract than (or a superconcept of) *Lake*, therefore, the probability of the former (0.00248) is greater than the probability of the latter (0.00003). As a result, the information content of *Water* (i.e., $-\log(0.00248) = 8.66$) is less than the information content of *Lake* (i.e., $-\log(0.00003) = 14.85$).

According to this approach, the similarity of hierarchically organized concept nouns is given by the maximum information content shared by the nouns, that is, the more information two nouns share, the more similar they are [8]. Note that given two nouns, say n_1, n_2 , the maximum information content shared by n_1, n_2 in the taxonomy is provided by the superconcept of n_1, n_2 whose information content is maximum (i.e., when defined, the least common superconcept). Starting from these assumptions, concept noun similarity is defined by the maximum information content shared by the nouns divided by the information contents of the comparing concept nouns (*information content similarity (ics)*) [6].

In our running example consider *Lake* and *Sea*. Their least common superconcept exists in the hierarchy and it is provided by *Water*. Therefore, the following holds:

$$\text{ics}(\textit{Lake}, \textit{Sea}) = \frac{2 \log p(\textit{Water})}{\log p(\textit{Lake}) + \log p(\textit{Sea})} = \frac{2 * 8.66}{14.85 + 11.18} = 0.67.$$

In the following, since concept intents are defined by sets of attributes, we will refer to attributes rather than nouns.

4. Similarity between FFCA concepts

In the following the notion of similarity between FFCA concepts is introduced. We have seen that a concept in FFCA is a pair formed by a fuzzy set of objects and a set of attributes. Similarity of FFCA concepts is computed by separately evaluating the similarity of fuzzy sets of objects and the similarity of sets of attributes and, successively, by combining them. They are presented below.

Regarding the similarity of concept extents, let us start by illustrating the notion of *similarity of fuzzy sets*, which is in line with [9] and requires first the notion of *fuzzy set cardinality*. The cardinality of a fuzzy set A in X , denoted as $|A|$, is given by the sum of the grades of membership of all the elements of X defined in A . For instance, the cardinality of the fuzzy set A :

$$A = ((H1,1.0),(H2,0.5),(H3,0.5),(H5,1.0))$$

is $|A| = 3$. Given two fuzzy sets A and B , their similarity, referred to as $SetSim(A, B)$, is given by the cardinality of the fuzzy set intersection divided by the cardinality of the fuzzy set union of A and B . For instance, consider the fuzzy set A above and the fuzzy set $B = ((H2,1.0),(H4,1.0),(H6,1.0))$. The similarity $SetSim(A, B)$ is defined as follows:

$$\begin{aligned} SetSim(A, B) \\ = \frac{|A \cap B|}{|A \cup B|} = \frac{|(H2,0.5)|}{|(H1,1.0),(H2,1.0),(H3,0.5),(H4,1.0),(H5,1.0),(H6,1.0)|} = \frac{0.5}{5.5} = 0.09 \end{aligned}$$

With regard to concept intents, we have seen they are represented as sets of attributes. The comparison between concept intents presented below has been inspired by the *maximum weighted matching* problem in bipartite graphs (for a formal presentation of the approach, refer to [4]). Informally, consider a lexical database for the English language, and two sets of attributes I_1, I_2 , of two fuzzy concepts that do not necessarily belong to the same context. Let a *candidate set of pairs* be a subset of $I_1 \times I_2$ such that there are no two pairs in the set sharing an element. For instance, assume that I_1 and I_2 represent a set of boys and a set of girls, respectively, a candidate set of pairs defines a possible set of marriages (when polygamy is not allowed). Within all possible candidate sets of pairs, consider (one of) the set(s) such that the sum of the values of the information content similarity (*ics*) of the pairs of attributes is maximal. Such a sum will be indicated as $\mathcal{M}(I_1, I_2)$.

For instance in our running example, assume $I_1 = \{SwPool, Sea\}$, and $I_2 = \{SwPool, Meal\}$. Within all possible sets of pairs of attributes that can be formed with I_1 and I_2 as described above, the set of pairs with maximal sum is the following:

$$\{(SwPool, SwPool), (Sea, Meal)\},$$

since $ics(SwPool, SwPool) = 1$, and $ics(Sea, Meal) = 0$. Therefore:

$$\mathcal{M}((SwPool, Sea), (SwPool, Meal)) = 1,$$

whereas the other possible set of pairs:

$$\{(SwPool, Meal), (Sea, SwPool)\}$$

leads to a null value (the *ics* of both the pairs are null).

Below the notion of similarity between FFCA concepts, referred to as *ConSim*, is presented. It is essentially given by the weighted average between the fuzzy set similarity of the concept extents $SetSim$, and the maximal sum $\mathcal{M}(I_1, I_2)$ above (up to a normalization factor). Formally,

given two fuzzy concepts $C_1 = (E_1, I_1)$, $C_2 = (E_2, I_2)$:

$$ConSim(C_1, C_2) = SetSim(E_1, E_2) * w + \frac{\mathcal{M}(I_1, I_2)}{m} * (1 - w)$$

where $SetSim$ is the fuzzy set similarity, $\mathcal{M}(I_1, I_2)$ is defined as above and m is the greatest between the cardinalities of the sets I_1, I_2 . Finally w is a weight, such that $0 \leq w \leq 1$, which is established by the domain expert.

Note that $ConSim$ is always a value between zero and one and, for any pair of concepts C_1, C_2 :

$$ConSim(C_1, C_2) = ConSim(C_2, C_1).$$

Consider our running example, and assume $w = \frac{1}{2}$. Let us first evaluate the similarity between two hierarchically related concepts in the Fuzzy Concept Lattice of Figure 2. For instance, consider:

$$\begin{aligned} C_1 &= (((H2, 1.0), (H4, 1.0), (H6, 1.0)), (SwPool)) \\ C_2 &= (((H4, 1.0), (H6, 0.8)), (SwPool, Sea)) \end{aligned}$$

We have seen that the fuzzy set similarity, $SetSim$, of the extents of the concepts is computed on the basis of the fuzzy set intersection and union as follows:

$$\begin{aligned} SetSim(((H2, 1.0), (H4, 1.0), (H6, 1.0)), ((H4, 1.0), (H6, 0.8))) &= \\ &= \frac{1.8}{3} = 0.6 \end{aligned}$$

Furthermore, $\mathcal{M}((SwPool), (SwPool, Sea)) = 1$. As a result ($m = 2$):

$$ConSim(C_1, C_2) = \frac{1}{2} * 0.6 + \frac{1}{2} * \frac{1}{2} = 0.55.$$

We observe that, in line with the proposal for non-fuzzy concepts given in [5], the information content approach leads to a fundamental difference with respect to other proposals, including [4]. This point, that will also be discussed in the Related Work Section, it is illustrated by the following example. Consider the fuzzy concept C_3 below, belonging to a different context, say *Italian Hotels*, containing the object $H7$ and the attribute *Lake* do not belonging to the context *Sardinia Hotels*:

$$C_3 = (((H2, 1.0), (H4, 0.7), (H7, 0.6)), (SwPool, Lake))$$

The similarity of this concept with, for instance, the concept C_2 above is computed as follows:

$$SetSim(((H2, 1.0), (H4, 0.7), (H7, 0.6)), ((H4, 1.0), (H6, 0.8))) = \frac{0.7}{3.4} = 0.20$$

Furthermore $\mathcal{M}((SwPool, Sea), (SwPool, Lake)) = 1.67$ since previously we have seen that $ics(Sea, Lake) = 0.67$. Therefore ($m = 2$):

$$ConSim(C_2, C_3) = \frac{1}{2} * 0.20 + \frac{1}{2} * \frac{1.67}{2} = 0.52.$$

Note that in [4], we had no way to automatically obtain the similarity degree between *Sea* and *Lake*. In fact, in that paper, the analysis performed by a panel of experts in the given application domain was needed, establishing axiomatic similarity degrees for attribute pairs. In this proposal the human expertise has been replaced by the notion of *ics* that makes use of lexical databases for the English language available on the Internet (in this example *WordNet*).

5. Related Work

Among the proposals for evaluating concept similarity in FFCA, we selected [9] where a framework for automatic generation of fuzzy ontologies from uncertainty information has been defined, named FOGA. In FOGA the notion of fuzzy formal concept similarity is essentially based on the similarity of concept extents $SetSim$. Therefore, with respect to $ConSim$ proposed in this

article, the approach adopted in [9] is restrictive since it focuses on the similarity of the concept extents, disregarding the similarity of the related intents.

An interesting proposal concerning similarity in Fuzzy Concept Lattices has been defined in [2], mainly to solve the problem related to the large number of concepts that can be extracted from data in a Fuzzy Concept Lattice. In particular, the similarity measure defined in the mentioned paper has been conceived for Concept Lattices, assuming that the intents and extents of concepts are strictly intertwined. Here, in line with [4, 5], the approach is oriented to the development of the Semantic Web and, in particular, is intended to support activities such as ontology mapping, integration, etc...where, in general, the intensional components of concepts are emphasized and can be defined without the extensional components [1].

As mentioned before, both [4, 5] focus on non-fuzzy FCA. Here we have addressed a proposal which extends [5] to fuzzy FCA, and which overcomes the limitations of [4] with regard to the dependency of the method on human expertise. In fact, in [4] the existence of a predefined domain ontology containing similarity degrees for any pair of attributes, defined in the application domain, is assumed. Such similarity degrees are axiomatically established by a panel of experts in the domain, according to a consensus system. Here, the axiomatic similarity degrees have been replaced with the information content similarity (*ics*) scores which can be computed without relying on human expertise. In fact, the *ics* can be automatically evaluated according to any lexical database for the English language (see for instance, in our running example, the *ics* between *Sea* and *Lake*).

With respect to other similarity measures proposed in the literature, such as *Dice*, or *Cosine* or *Jaccard* [7], a further contribution of this paper consists in the possibility of evaluating FFCA concept similarity by explicitly addressing the similarity scores of concept attributes. In fact, in the mentioned proposals, only the cardinality of the set of pairs of attributes showing affinity is considered, and the similarity scores of the pairs are not addressed. For instance, in our running example, the affinity of the pair (*Lake,Sea*) would count 0 or 1 rather than 0.67.

Regarding the choice of Lin's approach, it has been selected, in line with [5], since it shows a higher correlation with human judgement, than other methods for evaluating similarity within a taxonomy, e.g., Resnik, Wu&Palmer, etc..., and the traditional time-honored method to evaluate semantic similarity in a taxonomy referred to as *edge-counting* approach [6].

As a final remark, we observe that the evaluation of this proposal has been performed, on one hand, on the basis of theoretic considerations and, on the other hand, on the experimental results existing in the literature. A concrete experimentation has not been given here since, due to the inherently different underlying assumptions of the aforementioned proposals, it risks to have a low relevance. The only comparable proposal is [9] for which, as discussed above, concept similarity is restricted to concept extent similarity. In fact, the goal of the mentioned paper is the generation of a fuzzy ontology from uncertainty data rather than the similarity measure itself.

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