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**A MATHEMATICAL MODEL  
FOR ENERGY MARKETS**

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## Abstract

In this paper we present a centralized model for managing, at the same time, the day-ahead energy market and the reserve market in order to minimize, beside energy price, the overall cost of reliability and to assure that a given percentage of demand will be shielded by any failure situation without additional reservation costs. The model addresses, also, the very important point of long-term market efficiency, giving the right economic signals and incentives to guide investments in transmission as well as in production facilities. Finally, the model tries to overcome the negative aspects of existing centralized systems, namely, obscure price determination and locational energy prices, by determining a unique energy price (which reflects, though, externalities due to transmission and operational constraints) by the means of an auction scheme. The model is based on a mathematical optimization program and we propose a simple heuristic coupled with a *cutting plane algorithm* to solve it very quickly.

*Key words:* Energy Markets Design; Survivable Networks; Cutting Plane.



## 1. Introduction

Recent liberalization of the energy sector in many countries has brought forth market mechanisms for the efficient allocation of demand/supply. With this reform, the legislator had in mind the optimization of social welfare that, in simpler terms, translates in electricity lower price for citizens and in the effective use of resources. The legislator, in fact, believed that competition would fasten innovation in the production as well as in the transmission technology, attracting economic resources from the private sector. The subsequent utility industry restructuring has led to separate the Generation, Transmission and Distribution functions so that these activities are now owned separately and interact through the activities of independent organizations. In the USA market they are known as ISOs (Independent System Operators) or RTOs (Regional Transmission Organizations) and every country has a corresponding one (e.g. GRTN in Italy). These organizations do not own transmission or production assets: they operate or direct the operations of assets owned by their members. The reason for this is that the “power grid” is an incredible complex mechanism and needs to be operated in a centralized way. There are two main technical reason for this:

1. Electricity flows freely along all available paths from the generators to demand points in accordance with physics laws, dividing among all connected flow paths in the network, in inverse proportion of “impedance” on each path. Hence, flow can not be “easily” controlled as in the case of other network systems (telecommunications, water, gas, etc). The only way to control “flow” is by leveraging production (and/or demand).
2. The system needs to be “balanced” and “stable” at each point in time. Power generation and demand must be matched continuously because frequency needs to be the same within the system. Failure to do so causes equipment damages and above all, more or less extended, black outs. Stability translates, among other things, to limits on the maximum amount of energy that can be safely routed over transmission lines.

From the above it follows that this central organization, from now on referred to as the *Network Operator*, needs to direct and organize in some way the production of electricity. Its activities span real time operations as well as the planning phase. In real time operations its essential activity is to “follow the load” (i.e., continuously match energy supply and demand) and to “manage congestion” (that can occur due to scarce transmission capacity): it does so by increasing or decreasing energy production in real time. Also, when selecting which generators need to increase or decrease energy production the system operator needs to take into account production technical constraints such as ramp rates, capacity, minimum load, etcetera. In fact, not all generators can adjust easily and quickly their output. This essentially depends on the type of technology used to provide electricity. Due to these technical constraints the network operator acquires into the market, before actual operations, *Reserves* that are to be used to keep the system stable. These *Reserves* are not to be compared to *Inventories* since electricity can not be stored: they are instead *Capacity Reservations*, i.e., agreements with energy producers to supply a predetermined amount of energy if called in operation. Also, depending on the market structure, some *Network Operator* can rely on real time markets in which generators offer to increase or decrease energy production for a price. The network operator will choose in this market the offered *incs* and *decs* that can regulate the system at the minimum price. If a real time market of *incs* and *decs* is present, *Reserves* have the economic role of mitigating energy price volatility. In the planning phase the latitude of the *Network Operator* activities varies

according to the chosen market design. In any case, though, the *Network Operator* needs to assure that the scheduled energy production meets the transmission system constraints. The *Network Operator* is also responsible for very high system reliability. Its guiding principle is the so called “N minus 1 criterion”: “the system must be operated in such a way to remain secure upon failure of the most important component (generator or transmission line)”. In general, it is required to ensure a high degree of “survivability” for the many contingencies, more or less likely, that can happen. This implies that the network operator does not only need to real-time operate the grid but also needs to plan for the unexpected. The severe blackouts that were experienced recently in so many countries, though, have brought many to think that energy markets operating procedures need to be revised.

This operational complexity makes the design of efficient energy markets a real challenge. The market needs to pursue three main goals: efficient allocation of energy among suppliers and demanders, efficient allocation of transmission capacity, allocation of costs related to the grid operations (safety, reliability, power loss, etc.). From a more economic standpoint the market structure and its procedural rules need to be constructed carefully not only to avoid gaming and reach efficiency but also to allow effective price discovery. Also, the market need to be constructed and operated in such a way to give the right signals for long term efficiency, including investments in facilities for generation and transmission. It goes without saying that the market design and operational procedures need to support competition, ensuring that the right measures are in place to diminish dominant incumbents market power and promoting entry of newcomers.

If we look at the models for energy markets proposed in the literature and implemented worldwide we can make a first big classification: decentralized systems and centralized systems. In the former, implemented, for example, in California and Scandinavia (NordPool), there are usually several markets: a market for energy, a reserve market (ancillary services), a real time imbalance market, a congestion management market, etc., in which prices are set (mostly) as equilibria between supply and demand. Bilateral contracts are also possible. Moreover, several financial markets are possible (auctions for transmission rights, futures, hedges etc) as well. In decentralized systems, beside real time operations and safety, the *Network Operator* may be responsible for creating transmission rights to be auctioned at the market and in any case for guaranteeing feasibility (with respect to transmission and reliability constraints) of energy trades. In decentralized systems, markets are run in sequence: first, the energy market (bilateral contracts, exchange, day-ahead etc.) in which supply/demand allocation is pursued without any consideration of operational constraints (power loss, safety, reliability, etc). According to the specific market design, players might buy in the market transmission rights, that are usually financial obligations (coupled or not with service priority) to hedge against congestion charges. Even in the presence of transmission rights (that are auctioned on the basis of physical system capacity), market allocation may not be feasible for all the operative constraints. In this case the *Network Operator* needs to run a market to obtain a planned feasible production schedule. The market participants, suppliers and demanders, bids in *incs* and/or *decs* that represent the amount they are willing to increase or decrease, respectively, production or demand for the declared prices. Supply *incs* requires a payment from the *Network Operator* while supply *decs* require a payment to the *Network Operator* since they are seen as a deviation from previously scheduled (and paid for) energy production. On the contrary, demand *incs* require a payment to the *Network Operator* while demand *decs* requires a payment from the *Network Operator*. In order to select the least cost pattern to alleviate congestion the *Network Operator* might run an OPF or some similar optimization program. The “congestion charge” is the cost (or the

revenue) that the *Network Operator* needs to sustain (or to get) to alleviate congestion. If the cost is positive than a congestion charge is imposed to market participants, otherwise, if the cost is negative (i.e., the revenue is positive) than it is conveyed to the owners of transmission assets. This congestion charge could be applied uniformly to all players (demanders and suppliers) or only to those (energy injection and/or extraction points) causing the congestion. This latter approach is mostly implemented, causing different energy prices at different locations (nodes or zones). Once a feasible allocation is obtained the *Network Operator* needs to run a market for ancillary services. These are also known as *Reserves* and are usually required to be a percentage of the scheduled energy production. There are various type of *reserves*, (e.g., *spinning*, *not-spinning*, *black-start* etc.) differentiated by the time they can be available and sustained for. For example, *spinning reserves* need to be available within 10 minutes and sustained for two hours. Other reserves are needed to abide the reliability constraints set by regulatory bodies (e.g., NERC in USA). In centralized systems, instead, the *Network Operator* optimizes the allocation of energy subject to operational constraints. Procedural rules varies a lot in practical implementation but, in theory, the *Network Operator* should receive supply and demand curves (possibly constrained: e.g., minimum load, maximum capacity, maximum price, etc.) and, keeping in consideration all system and operative constraints, should find the least cost allocation that optimize each player utility function. Centralized systems can still allow bilateral contracts but special care need to be given to the transmission allocation procedure to avoid unfair treatment of all involved players. In most practical implementations not all operational constraints are considered at the same time and, also, some constraints get linearized. For example, some optimize just energy allocation subject to network linear constraints without taking into consideration reliability and/or reserves, others optimize allocation for the single hour, without taking into consideration inter-temporal constraints, etc. This is to avoid that the corresponding optimization program becomes intractable or solved too slowly with respect to market needs. Also, few centralized market have implemented demand-side bidding and most rely on point predictions of demand, so the optimization aims to minimize the total cost of serving the predicted demand net of the portion included in bilateral contracts. There is a good economic reason for that, though, as we will see shortly, as well as the normal strive towards simplicity. Usually, in the end, the optimization program that is solved is a linear program. The solution to this program gives the optimized energy dispatch while energy prices are determined as the shadows prices at each node. Hence, this method of determining prices is called “nodal pricing” and it reflects the cost of producing energy as well as transmission charges and possibly other costs related to reliability and security. If only the transmission constraints are considered then the price difference between two consecutive nodes is the “transmission congestion rent” that is to be assigned to the transmission link owner. Note that, if a line is not congested, the node price at its endpoints is the same and the congestion rent is zero. The nodal price method is deemed necessary because it is not possible to use explicit transmission capacity price curves in the optimization program since, as previously said, transmission capacity usage is dictated only by physics laws. Finally, very few markets have implemented systems that minimize simultaneously the costs of allocating energy and reserves subject to transmission constraints. Some pools, like for example the New England pool (NEPool), require all suppliers to be available at real time to provide additional energy or curtailment (up to capacity limit) if needed, but they aren’t paid for this availability. Most of the time, the *reserves* plan is made after the energy market closes even in centralized systems. Also, the N-1 criterion reliability analysis is a static analysis that is pursued after a feasible energy production schedule is available.

If we would like to compare the two systems, decentralized and centralized, it is clear that, from a pure optimization standpoint, a centralized system in which all operative constraints are considered at the same time gives the “best” solution. Moreover, it avoids “gaming” that can easily happen when running sequential markets. For example, a supplier with a location advantage will withhold some of its capacity from the day ahead energy market if it can profit more in subsequent markets. This strategic behavior can create artificial system congestion or an energy price sky-rocket increase in the real-time spot market. This is actually what happened in California when its decentralized energy market started operations. To avoid gaming various counter-measures can be taken. For example, in Scandinavia (NordPool) price bid in subsequent market are carried over from the day ahead energy market. The advocates of decentralized systems, though, support the idea that gaming can be suppressed by the right procedural rules and that the optimal dispatch of an optimization program is usually just an approximation to the real optimal solution with no real guarantee that energy prices paid by demanders will be lower than those obtained in a decentralized system. The real strength, though, of decentralized system supporters is in the claim that the price formation in tight power pool is too obscure and doesn’t give the right economic signals to market participants, neither in the primary (bidding) nor in the secondary (financial) markets. The separation among the various markets, in fact, makes clear the cost component of each resource in the determination of the final price. Also, a separate energy market allows the determination of a unique energy price that clears the market. The determination of a single price for energy is important if one is to construct a secondary market for forward, futures, hedges, etc. The point made by decentralized system advocates of the need of a single energy price that clears the market is important but this by itself doesn’t disqualify centralized systems. In fact, we will shortly present a centralized system model in which a single price is determined. Also, the model addresses the problem of giving the right incentives to market players to reach long-term efficiency, in particular regarding transmission utilization. In the current models, both centralized and decentralized, transmission asset owners receive the “congestion rent” only when the transmission capacity is used up to its limit. This scheme doesn’t give any incentive to transmission assets owners to install more capacity. Actually, it gives incentives to remove capacity! Finally, some consideration about price determination. In decentralized systems the energy price is determined as an equilibrium price between supply and demand. Following Hicks, we can say that it is a *flex-price* system. This is the way prices are determined in basically all commodity markets. But, even if electricity can be seen from many point of views as a commodity, it has a very small demand elasticity which becomes practically zero in the day-ahead and hour ahead market. Small consumers (that make up most of the electricity consumption) are barred by directly bidding in energy markets and large consumers (e.g., industrial facilities) can not economically change their production schedules (that consume energy) in the time frame of a day or few hours ahead. Hence, the demand bids in the market are from retailers and distribution companies and they do not reflect the true utility functions of final consumers. Retailers and distributors derived demand is anyway quite inelastic. As the time horizon increases, though, demand can become more price sensitive. In this case, though, long-term bilateral contracts (either privately managed by the parties or sold through an exchange) are best suited to pursue demand interests. But actually they are also best suited to pursue supply interests, especially those of energy producers whose optimal dispatch configuration requires to generate energy continuously at the same rate for consecutive hours. The day-ahead and hour-ahead markets are, hence, best suited for a *fix-price* system in which the price is set by sellers (suppliers) on the basis of their own costs. The appropriate market design is then that of an auction among all suppliers in order to allocate the

energy demand (known or predicted). Various type of auction can be used (and there is a vast literature about electricity auction design), all of them having as an output the allocation of energy demand at a single price.

## 2. Survivable energy markets: a comprehensive model

In this section we will present a centralized model for the day-ahead (or hour-ahead) market that tries to overcome the negative aspects of existing centralized systems (namely, obscure price determination and locational energy prices). Moreover, it will explicitly consider reliability costs, both hidden and not-hidden, in the determination of the optimal dispatch. In order to do so, the *energy day-ahead market* and the *reserve market* are merged and run together. The proposed model should minimize, among others, the overall cost of *reliability reserves* as well as the chances of extended blackouts due to unforeseen events. Finally, a scheme is proposed to reward transmission assets owners consistently, giving them the right incentives to upgrade capacity, if needed.

The proposed market model is a constrained auction, in which suppliers bid in their hourly supply curve as well as their capacity reservation prices and the *Network Operator* selects, among all the bids and among all the bilateral contracts production schedules, the production pattern that will satisfy demand at the minimum overall cost while abiding to operative constraints and will determine an optimal reservation plan. A single energy price will be determined that does, though, reflect externalities due to transmission and other operative constraints. The *cost of reserves* is kept distinct, though, from the *energy price*. A transmission charge will be collected from each market player (demonder and supplier) proportionally to the used transmission capacity and distributed to transmission assets owners proportionally to their effectively used capacity. This scheme rewards transmission assets owners proportionally to the amount of the owned capacity that is effectively used to transmit energy and not for its utilization (proportion of capacity used). If a transmission link is congested the owner doesn't get an additional benefit but the link capacity limit becomes an obstacle for additional revenue stream. Assets owner, hence, have the incentive to upgrade capacity, if needed. Beside transmission constraints (Kirchoffs laws, capacity limits, etc.) the model explicitly considers the cost of reliability, represented by the "N minus 1 criterion". Most implemented systems do an ex-post, static analysis of the reliability constraints: given an energy flow, they simulate system behavior when a single link fails, i.e., they will check if the flow will safely redistribute over remaining lines upon the failure of any single transmission link. If this feasibility check fails, ad hoc procedures are undertaken to get a feasible, reliable energy flow. Once a feasible energy flow is obtained, the *Network Operator* will still need to reserve capacity in the *Reserve Market* to face the event that a single supplier, scheduled for production, is cut off from the grid. This can happen if, for any reason, all of a sudden, the supplier can not provide all or part of the scheduled energy or if a transmission line cuts off the supplier from the demand locations. The cost of *Reserves* is paid by all market players and, in the end, by the final consumer. As we will show, appropriate production schemes, that redistribute production load among various network locations, can significantly reduce the amount of this type of reserves. The model we will present computes the tradeoff between the simple reservation scheme and the production leverage scheme, resulting in a least cost solution that will tell the optimal amount each supplier will produce (paid at or above the supplier bid price) and the optimal amount of capacity to reserve in the *Reserve Market* to abide to reliability constraints. Also, a given percentage of the total demand will be shielded by the risk of blackouts without the need of securing reserves.

Let us detail the model, now.

Demand, at each network location, is predicted and decomposed in two components: the part covered by bilateral contracts and the part to be covered by bids. We will have, hence, for each demand location  $i$ ,  $i = \{1, \dots, n\}$ ,  $d_i = d_i^{cov} + d_i^{bid}$  where  $d_i^{cov}$  represents the total amount of demand at location  $i$  covered by bilateral contracts while  $d_i^{bid}$  represents the total amount of demand at location  $i$  to be covered by accepted bids in the market. Bilateral contracts do not have priorities over transmission lines capacity usage: they are treated as bids with zero unit price up to the amount agreed in the contract. If there are  $r$  suppliers that entered bilateral contracts and  $b_i$ , for  $i = \{1 \dots r\}$ , is the total amount supplier  $i$  has agreed to supply at the terms specified in the contract with each of its customers then it could be scheduled by the *Network operator* to deliver up to the amount  $b_i$ . This supplier could, obviously, bid in the market its residual capacity, if any. If supplier  $i$  is scheduled by the *Network operator* to supply less than the contracted amount  $b_i$ , then he needs to pay the *Network operator* a corresponding proportion of its contracts revenue. For example, if  $x\%$  of  $b_i$  is not scheduled and he holds  $s$  bilateral contracts, each with a unit price of  $p_j$  then he needs to pay the *Network Operator* an amount  $P = \sum_{j=1}^s x\%b_i(j)p_j$ , where  $b_i(j)$  is the amount the supplier  $i$  contracted with its customer  $j$  to be paid at the unit price of  $p_j$ . Similarly, if  $p$  is the “market price”, customer  $j$  will pay the *Network Operator* the amount  $x\%b_i(j)(p - p_j)$  if  $p \geq p_j$ , otherwise he will get from the *Network Operator* the amount  $x\%b_i(j)(p_j - p)$ . If supplier  $i$ , depending on its market bid, is scheduled to supply more than  $b_i$  then he will be paid the “market price”  $p$  for this additional supply. Hence, bilateral contracts are treated here as *Contracts for Difference* or CFD, as they are known. Other schemes are obviously possible but this is the prevailing scheme in most energy markets and it guarantees non-discriminatory access to the transmission grid.

Suppliers bid in, each hour, their hourly supply curve as well as their capacity reservation prices, constrained only by capacity limits (minimum load, maximum capacity). Here we assume that the market is cleared, sequentially, every hour, independently of bids submitted in different hours. This is customary in basically all the commodity markets and is the design adopted in most energy markets, except that, in the day ahead market, suppliers usually bid in their hourly supply curve for all the hours at the same time. Prices sets in the day-ahead market are binding so that the real-time spot market is used only to price small imbalances in demand-supply and to manage real time congestion. Also, one may think of carrying over prices from the day ahead market into the real time market to avoid supplier gaming. In the optimization model we propose, the *Network Operator*, in its selection procedure, will not take in consideration other supplier constraints (such as ramp rates, sustained duration of the load, etc.) in the view that each supplier can optimize its own production schedule by entering in bilateral contracts and by adjusting bids in subsequent hours. Even though the optimization model to be used by the *Network Operator* to clear the market, subject to operative constraints, could easily handle, as we will see, these additional supplier constraints, our view here is that the proposed scheme should give more incentive to suppliers with complex production and inter-temporal constraints (such as thermal generators) to be more active in the forward market of bilateral contracts and in general in the secondary market, contributing hence to its depth and liquidity. Each producer supply curve,  $P(q)$ , indicates the minimum unit price that it is willing to accept for producing an amount  $q$  of energy. If the *Network Operator* selects, among all the bidders,  $m$  suppliers to deliver, respectively, an amount  $q_i$  of energy then the market clearing price  $p$  is equal to the maximum unit price selected i.e.  $p = \text{MAX}_{\{i \in I\}} P_i(q_i)$  where  $I$  represents the set of selected suppliers. In the extreme case (very unlikely) in which the *Network Operator* selects only bilateral contracts to cover energy demand, no real market clearing price exists.

For practical reasons, one could use the highest unit price set in bilateral contracts or, better, a weighted average of these prices. Beside supply curves, energy producers (also those holding bilateral contracts) bid in call prices for *availability*. Here  $R(q)$  indicates the unit price that a supplier needs to be paid for reserving capacity up to the amount  $q$  in the *day-ahead market*. These availability prices are paid regardless the supplier's actual energy delivery in real time operations. Suppliers that are active in the *reserve market* bid in reservation prices  $S(q)$  to be used in the case they are included in the *reservation plan*.

Having received all the bids and bilateral contract schedules the *Network Operator* selects the optimal dispatch as well as the optimal reservation plan by solving a constrained optimization problem.

Let us indicate with  $h_i \geq 0$  the variable amount of energy to be produced by supplier  $i$  according to its bid and with  $g_i \geq 0$  the variable amount of energy to be produced by supplier  $i$  up to the limit  $b_i$  contracted in bilateral contracts. For ease, let us introduce the variable  $f_i = h_i + g_i$  which represent the total variable amount of energy produced by supplier  $i$ . Let us indicate with  $x_i \geq 0$  the variable amount of capacity reserved at supplier  $i$  in the *day-ahead market* and with  $y_i \geq 0$  the variable amount of capacity reserved at supplier  $i$  in the *reserve market*. Usually, capacity is reserved in integer amounts so that  $x_i$  and  $y_i$  are required to be integer. As we know, there are different types of *reserves* (spinning, not spinning, etc.), hence we should distinguish among them by adding a variable for each type and for each supplier. For the sake of simplicity, we will assume that there is only one type of *reserve*, keeping in mind that all the results hold true (with the obvious modifications) when all the different types of *reserves* are modelled in the optimization program.

Let be  $D = \sum_j d_j$  the total amount of energy demanded across all locations. One of the constraint to be added to the optimization program is, hence, the one equating supply and demand:

$$\sum_{i=1}^n f_i = D$$

where  $n$  is the total number of energy producers in the *day-ahead market*.

Also, we need to add, for each supplier, the supplier's "availability" constraint,

$$f_i \leq x_i$$

and the supplier's capacity constraints,

$$m_i z_i \leq f_i \leq z_i C_i$$

where  $m_i \geq 0$  is the supplier  $i$  "minimum load" and  $C_i \geq 0$  is its maximum capacity.  $z_i \in \{0, 1\}$  is a binary variable that we'll take the value of 0 if supplier  $i$  is not selected in the optimal dispatch and 1 otherwise. We could as well embed these capacity constraints in the bid functions by setting a huge price in the range  $[0, m_i[$  and  $]C_i, \infty]$ , so that the optimization program will automatically drive out these values as possible solutions. This is actually what we will assume, at least for the minimum load requirement, to avoid the use of the decision variable  $z_i$ :

$$P_i(q_i) = \infty \quad \forall q \in [0, m_i[$$

The same hold true for the reservation prices i.e.  $R_i(q_i) = \infty \quad \forall q \in [0, m_i[$  and  $S_i(q_i) = \infty \quad \forall q \in [0, m_i[$

10.

Hence, each supplier capacity constraint simplify to:

$$x_i \leq C_i \quad \text{and} \quad y_i \leq C_i$$

As we said, we can easily model other supplier's constraints. For example, if a supplier can't adjust its production schedule in real time then we may add the constraint that

$$\lceil f_i \rceil = x_i$$

or if he can increase its production schedule only in a given range from its scheduled production level then we may write that

$$x_i \leq \lfloor f_i + r(f_i) \rfloor$$

where  $r(f_i)$  is a value depending on the flow value  $f_i$ .

We can now add the reservation requirement:

$$\sum_{i=1}^n x_i + \sum_{i=1}^m y_i = \lceil D + \rho\%D \rceil$$

where  $\rho\%$  is the reservation parameter. Under our assumption of one single type of reserve, this constraint just says that the total amount of capacity reserved, both in the day-ahead market and in the reserve market is equal to the total demand plus a given percentage of it. Let us call  $H = \lceil (1 + \rho\%)D \rceil$ .

Let us indicate by  $A$  the generation plan feasible set where feasibility is checked against all transmission constraints, i.e.,  $F = (f_1, f_2, \dots, f_n) \in A \iff F = (f_1, f_2, \dots, f_n)$  satisfies all transmission constraints, particularly the capacity limit at each line.

Let us indicate by  $A^t$  the generation plan feasible set where feasibility is checked against all transmission constraint in a network, though, in which the link  $t$  has been removed. Hence  $F = (f_1, f_2, \dots, f_n) \in A^t \iff F = (f_1, f_2, \dots, f_n)$  satisfies all transmission constraints in  $N^t = (V, E - \{t\})$ , where  $N^t$  represent the transmission grid without the transmission line  $t$ .

The last constraint partly accounts for the "N-1" criterion but it is not enough. The *Network Operator* needs to assure that enough capacity is reserved to face the contingency that an entire generator is cut off from the grid. Hence, it could add, for each generator  $j$ , the following constraint

$$\sum_{i \neq j}^n x_i + \sum_{i=1}^m y_i \geq \lceil D \rceil$$

that assure that, in the case generator  $j$  is cut off from the grid, there is enough reserved capacity to cover the demand  $D$ . But we can actually express better the trade off between load balancing and reserve requirement.

By adding, for each supplier  $j$  in the day ahead market, the constraint:

$$\sum_{i \neq j} f_i \geq L$$

we assure that, if any supplier is cut off from the grid, there is still, at least, an amount of flow  $L$  circulating in the grid, so that, at most, the amount  $\lceil D - L \rceil$  needs to be reserved in the *Reserve Market*. The higher the value of  $L$  the lower the amount one needs to reserve in the reserve market. There is, hence, a clear trade-off between load balancing across location (even if this may imply scheduling for dispatch more expensive generators) and reserving capacity in the

*Reserve Market.*  $L$  can be a variable set in the optimization program, by adding, for example, the constraint  $L = D - \sum_{i=1}^m y_i$  or it may be a parameter derived by physical and reliability constraints. For example, when a generator is cut off from the grid the reserved energy needs to be available basically in real time in order to avoid blackouts. If the total available capacity in the market for this type of reserve is  $T$  then it has to be the case that  $L \geq D - T$ . In general, if  $L$  is the amount that *survives* any failure situation then  $\frac{L}{D}$  is the percentage of total demand that is automatically shielded by possible blackouts, at least theoretically.

Finally, the *Network Operator* needs to assure that the reservation plan satisfies the transmission constraints. Let us indicate by  $F^j = (y_1, \dots, y_m, x_1, \dots, x_{j-1}, x_{j+1}, \dots, x_n)$  all the possible flow patterns that can be created by using the reserved capacity in order to satisfy demand  $D$  when supplier  $j$  is cut-off from the grid. Then it has to be that at least one of this flow pattern satisfies transmission constraints. For brevity we may simply write  $F^j \in A$ .

In the end the *Network Operator* will solve the following optimization program:

$$\text{Min} \sum_{i=1}^m S_i(y_i)y_i + \sum_{i=1}^n (R_i(x_i)x_i + P_i(h_i)h_i)$$

s.t.

$$h_i + g_i = f_i \quad \forall i = \{1 \dots n\} \quad (1)$$

$$g_i \leq b_i \quad \forall i = \{1 \dots n\} \quad (2)$$

$$y_0 = \sum_{i=1}^m y_i \quad (3)$$

$$\sum_{i=1}^n f_i = D \quad (4)$$

$$\sum_{i \neq j}^n f_i \geq L \quad \forall j = \{1 \dots n\} \quad (5)$$

$$y_0 + \sum_{i \neq j}^n x_i \geq \lceil D \rceil \quad \forall j = \{1 \dots n\} \quad (6)$$

$$\sum_{i=1}^n x_i + y_0 \geq H \quad (7)$$

$$f_i \leq x_i \quad \forall i = \{1 \dots n\} \quad (8)$$

$$x_i \leq C_i \quad \forall i = \{1 \dots n\} \quad (9)$$

$$y_i \leq C_i \quad \forall i = \{1 \dots n\} \quad (10)$$

$$F = (f_1, \dots, f_n) \in A \quad (11)$$

$$F = (f_1, \dots, f_n) \in A^t \quad \forall t \in E \quad (12)$$

$$F^j = (y_1, \dots, y_m, x_1, \dots, x_{j-1}, x_{j+1}, \dots, x_n) \in A \quad \forall j = \{1 \dots n\} \quad (13)$$

$$f_i \geq 0; g_i \geq 0; h_i \geq 0; x_i \geq 0; y_i \geq 0 \quad \forall i = \{1 \dots n\} \quad (14)$$

$$x_i, y_i \quad \text{integer} \quad \forall i = \{1 \dots n\} \quad (15)$$

By solving this program the network operator will get a least cost feasible production plan  $F = (f_1^*, \dots, f_n^*)$  as well as a reservation plan  $Res = (x_1^*, \dots, x_n^*, y_1^*, \dots, y_m^*)$ . Let be  $N^*$  the set of suppliers scheduled for production in the day ahead market, i.e.,  $f_i^* > 0$  for all  $i \in N^*$  and

$f_i^* = 0$  for all  $i \notin N^*$ . Also, let be  $Q^* \supseteq N^*$  the set of suppliers in the day ahead market included in the reservation plan and let  $M^*$  be the set of suppliers in the reserve market included in the reservation plan, i.e.,  $x_i^* > 0$  for all  $i \in Q^*$ ,  $y_i^* > 0$  for all  $i \in M^*$  and  $x_i^* = y_i^* = 0$  otherwise. We can set the price for energy,  $p$ , as  $p = \text{Max}_{\{i \in N^*\}} \{P_i(h_i^*)\}$  ( we remind the reader that  $f_i^* = g_i^* + h_i^*$ ).

Now, let us run, by using the same energy bids, an unconstrained (with respect to transmission and reliability constraints) auction to fulfill demand  $D$  at the minimum price. Let  $T^*$  be the optimal set of suppliers. If  $T^* = N^*$  then the operative constraints aren't binding. The selected suppliers are the most efficient in producing energy and other suppliers need to reduce cost and/or invest in more efficient technologies if they want to compete. If, though,  $T^* \neq N^*$  then  $N^* - T^*$  will indicate those suppliers that have a competitive advantage due to their position in the network. This information could be used to guide investments in more efficient production facilities in these locations so that in the long run we should have  $T^* = N^*$ . We may as well run the optimization program in which, though, all transmission capacity limits have been removed. Let be  $V^*$  the set of suppliers selected in this case for dispatch. Again, if  $V^* \neq N^*$  then transmission constraints are favoring less efficient suppliers. By analyzing the transmission line flow one can easily see how much capacity needs to be added and in which lines in order to have  $V^* = N^*$ .

Similarly, we could use the same pricing scheme for *reserves*, namely,

$$r = \text{Max}\{\text{Max}_{\{i \in N^*\}} R_i(x_i^*), \text{Max}_{\{i \in M^*\}} S_i(y_i^*)\}$$

or we could pay each selected supplier the amount it tendered.

Each demand node will be charged, hence,  $p * \hat{d}_i^{bid} + r * (1 + \rho\%)d_i$ , where  $\hat{d}_i^{bid}$  is the actual demand fulfilled at location  $i$ , net of bilateral contracts which will be finalized as CFDs according to the scheme previously outlined. Each supplier  $i$  will receive, accordingly,  $r * x_i^*$  or  $r * y_i^*$  plus  $p * \hat{h}_i^*$  where  $\hat{h}_i^*$  is the actual energy delivered by supplier  $i$ , net of bilateral contracts. Finally, let be  $t_e$  the unit cost for using transmission capacity. Each demand node  $j$  will be charged the amount  $\frac{1}{2}t_e * \hat{d}_j$  where  $\hat{d}_j$  is the actual demand realized at operation at location  $j$  while each supplier  $i$  will be charged  $\frac{1}{2}t_e * \hat{f}_i$  where  $\hat{f}_i$  is the amount of energy flow actually dispatched by supplier  $i$  at operations. Any congestion will be managed in real time by the *Network Operator* and its cost will be evenly distributed among all market players. The owner of the transmission link  $e$  will receive  $t_e * \hat{f}_e$  where  $\hat{f}_e$  is the actual flow on the transmission link  $e$ . Its owner, hence, can receive at most  $t_e * c_e$  where  $c_e$  is the link capacity. Limits on capacity are, hence, a limit to additional revenue stream. This reward scheme gives the right economic incentives to transmission assets owners to upgrade capacity if needed. The previous optimization model can be used to carry simulations to help investors decide which is the most profitable amount of capacity that needs to be added and where.

To summarize, the proposed model tries to overcome the negative aspects of centralized models and poses itself, hence, as a superior alternative to decentralized models as well. It doesn't use the nodal pricing scheme, as existing centralized energy market do, but an auction based scheme. The final energy price as well as the reliability cost are deduced by actual bids and this helps market players to better define their competitive profile and bidding strategies. There is a single unit price for energy (that reflects, though, externalities due to transmission and operational constraints) and one can opt as well as for a single unit price for reserves (auction scheme) or for pay-for-bid reservation prices (minimum cost). Bilateral contracts are allowed and finalized as CFDs. The proposed model merges together the *day ahead market* and the *reserve market* and basic optimization theory tells that this scheme is more cost efficient than running the two

markets in sequence, as it is done in most energy markets (both centralized and decentralized). Also, by merging these two markets, one avoids possible gaming from suppliers that could, otherwise, hold capacity to be resold later, at higher prices, in subsequent markets. Moreover, if the *day-ahead market* is run first and then the *Network Operator* reserves capacity in the *reserve market* according to the optimal dispatch obtained in the energy market, some suppliers could be unfairly favored in the selection simply because of their location in the network. This unwanted competitive advantage can allow them to bid in prices much above their actual marginal cost. Altogether these elements heavily support the idea that an efficient and economically sound energy market design needs to merge somehow and run at the same time the *day-ahead energy market* and the *reserve market*. Finally, the simple transmission pricing scheme proposed in the model gives transmission assets owner the right economic incentives to upgrade capacity, if needed. In the long run, the proposed model should guarantee that investments in transmission facilities as well as in more efficient energy production technology will be pursued. As of today, none of the existing models has completely addressed the very important point of the market long term efficiency.

### 3. Solving the model: a polyhedral approach

The last (but not least) point that needs to be addressed is how to solve the optimization program underlying the proposed market model. Detractors of centralized models often stress the fact that the actual “optimal” solution obtained by solving complex, constrained optimization models, is usually only an “approximation” of the real optimal solution so that the supposed superiority of this solution to the one obtained through the typical market mechanisms in place in decentralized models is all to be proved. Also, the market clearing mechanisms of decentralized models are extremely fast in providing solutions, and speed is a key element in this environment where market is cleared every hour. Hence, we need to assure that we are able to solve the proposed optimization program very quickly and that the obtained solution is optimal or very close to it.

First of all, note that any real function  $P(q)$  defined over the range  $[m, C]$  can be approximated, in each interval  $[q_i, q_{i+1}] \subset [m, C]$ , by a constant value  $p_i$ , that can be, for example the average value of  $P(q)$  over the interval  $[q_i, q_{i+1}]$  (or the minimum, the maximum, the median value etc.). For continuous functions, the smaller the interval width the better the approximation. Let’s assume that  $w$  is the number of disjoint intervals in which we have partitioned  $[m, C]$ . If we want to solve the problem:

$$\text{Min } P(q) * q$$

s.t.

$$\begin{aligned} q &\in S \\ 0 &\leq m \leq q \leq C \end{aligned}$$

where  $S$  is defined by a finite number of constraints, we could find an approximate solution by solving the problem:

14.

$$\text{Min} \sum_{i=1}^w p_i * x_i$$

s.t.

$$\begin{aligned} x_i &\in S \quad \forall i = \{1 \dots w\} \\ q_i z_i &\leq x_i \leq q_{i+1} z_i \quad \forall i = \{1 \dots w\} \\ \sum_{i=1}^w z_i &= 1 \\ z_i &\in \{0, 1\} \quad \forall i = \{1 \dots w\} \end{aligned}$$

Also, we can set  $q_1 = 0$  and  $q_2 = m - \epsilon$  by defining  $p_1 = \infty$  or a suitable big number.

In basically all practical applications, the bid functions are already given as step functions, i.e., constant over disjoint intervals so that we can solve our optimization program by solving an equivalent problem whose objective function is linear. Moreover, in many applications, we can also drop the  $z_i$  variables since, most of the time, we can prove that the optimal solution will fall anyway in a single interval.

Hence, from now on, under very mild assumptions, we will assume, for the sake of simplicity, that the bid functions are linear functions. Also, since our purpose here is to find the cheapest allocation of energy supply and reserve, we can linearize, as it is done in all (to the best of our knowledge) centralized energy markets, the transmission constraints.

The problem we want to solve, hence, will be a Mixed-Integer Program i.e., an optimization program with a linear objective function and linear constraints except for the requirement that some variables need to be integer.

$$\text{Min} \sum_{i=1}^m s_i y_i + \sum_{i=1}^n (r_i x_i + p_i h_i)$$

s.t.

$$h_i + g_i = f_i \quad \forall i = \{1 \dots n\} \quad (16)$$

$$g_i \leq b_i \quad \forall i = \{1 \dots n\} \quad (17)$$

$$y_0 = \sum_{i=1}^m y_i \quad (18)$$

$$\sum_{i=1}^n f_i = D \quad (19)$$

$$\sum_{i \neq j}^n f_i \geq L \quad \forall j = \{1 \dots n\} \quad (20)$$

$$y_0 + \sum_{i \neq j}^n x_i \geq [D] \quad \forall j = \{1 \dots n\} \quad (21)$$

$$\sum_{i=1}^n x_i + y_0 \geq H \quad (22)$$

$$f_i \leq x_i \quad \forall i = \{1 \dots n\} \quad (23)$$

$$x_i \leq C_i \quad \forall i = \{1 \dots n\} \quad (24)$$

$$y_i \leq C_i \quad \forall i = \{1 \dots n\} \quad (25)$$

$$F = (f_1, \dots, f_n) \in A \quad (26)$$

$$F = (f_1, \dots, f_n) \in A^t \quad \forall t \in E \quad (27)$$

$$F^j = (y_1, \dots, y_m, x_1, \dots, x_{j-1}, x_{j+1}, \dots, x_n) \in A \quad \forall j = \{1 \dots n\} \quad (28)$$

$$f_i \geq 0; \quad g_i \geq 0; \quad h_i \geq 0; \quad x_i \geq 0; \quad y_i \geq 0 \quad \forall i = \{1 \dots n\} \quad (29)$$

$$x_i, y_i \quad \text{integer} \quad \forall i = \{1 \dots n\} \quad (30)$$

Mixed Integer Programs, or MIP for brevity, are sometimes hard to solve through general purpose algorithms (and, hence, through commercial code for MIP problems) and to get an optimal solution may require more time than is granted in energy markets. Hence, we may think of getting an approximate solution, if, though, this solution is not too far away from the optimal one. Note that even a small difference between the optimal solution and the approximate one may be a significant value in absolute terms. After all, the optimal solution value is the energy and reservation cost of an entire electrical network!!!

Approximate solutions to MIP problems can be obtained, for example, by ad hoc heuristics. First, a point, optimal for a relaxation of the MIP problem, is obtained and then, applying the heuristic, a feasible point to the MIP problem is obtained, whose objective value is an upper bound (in the case of a minimization problem) to the optimal value. The gap between the feasible solution value and the relaxed solution value is an upper bound to the true gap between the value of the feasible solution obtained through the heuristic and the value of the optimal MIP solution. The smaller this gap the better the approximation. A relaxation of the MIP problem can be obtained by relaxing some constraints, or, in general, by optimizing the objective function over a set that contains the set of feasible points. The optimal value of the relaxed problem is a lower bound (in the case of minimization problem) to the optimal MIP value. If the solution obtained from the relaxed problem satisfies all MIP constraints, in particular the integrality constraints, then this is the optimal solution to the MIP problem. Otherwise, we may use some heuristic to find, starting from the relaxed optimal solution, a “good” solution, i.e., one that satisfies all MIP constraints and whose objective value is not too far away from the problem optimal value. The most used relaxation in practical applications is the linear relaxation, i.e., that obtained by dropping the integrality requirements. The reason is that Linear Programs, even those with many variables and constraints, are quickly solved by commercial codes. Hence, in our case, we would be very happy to solve our problem by solving a suitable linear program. Also, one of the most used heuristic, in practice, to get a feasible solution is rounding each not integer variable to the nearest integer, if feasible. For our problem, rounding each variable required to be integer to its ceiling, i.e., the smallest integer greater than its value, is always feasible under the assumption, met in all electrical networks planning models, that small variations in a supplier production output are feasible towards transmission constraints, and that each supplier maximum capacity is an integer value in the mathematical model. This rounding procedure, however, most of the time, will not give the optimal MIP solution, as it is confirmed by our testing. On average, this procedure provides a solution whose objective value differs by the optimal value by 0,58% with a maximum of 1% and a minimum of 0,02%. These percentages can be misleading, since they are referred to the whole electrical grid cost, i.e., big numbers, and where usually energy cost is much larger than the reservation cost. Hence, in many cases, the total energy cost in both the LP relaxed solution (and hence the rounded solution) and the MIP solution is the same. Should we consider, hence, the percentage difference between

the reservation cost of the rounded solution and the reservation cost of the optimal solution we would get about 1,60% with a minimum of 0,88% and a maximum of 2,83%. We need to find a better approach. The chosen approach is to find a better relaxation through a *cutting plane algorithm* and to apply a more sophisticated rounding heuristic to the relaxed solution. In our testing, the proposed method has always yielded, except one case, the optimal MIP solution.

We remind here the reader that a *cutting plane algorithm* is an iterative procedure that adds, at each round, one or more valid inequalities for the set of MIP feasible points that cut off the current optimal solution. Hence, at each iteration, a tighter relaxation is solved and the optimal relaxed value gets closer to the MIP optimal value. If the relaxed solution happens to satisfy all MIP constraints then it is indeed the optimal MIP solution. In order for a cutting plane to perform well, we better have on hand a set of valid inequalities for the set  $S$  over which we wish to optimize, which it happens to be a polyhedron. In [2] we have studied the structure of two polyhedra, referred to in the sequel as  $F_n(L)$  and  $Q_{n+1}(D, H)$ , which are defined by a subset of the linear inequalities defining  $S$ .

The polyhedron  $F_n(L)$  is defined by the following set of inequalities:

$$\begin{aligned} \sum_{i \neq j} f_i &\geq L & \forall j = \{1 \dots n\} \\ x_i &\geq f_i & \forall i = \{1 \dots n\} \\ x_i \in Z_+ & f_i \geq 0 & \forall i = \{1 \dots n\} \end{aligned}$$

i.e., inequalities (20),(23), (29) and (30) of our model.

while  $Q_{n+1}(D, H)$  is defined by:

$$\begin{aligned} x_0 + \sum_{i=1}^n x_i &\geq H \\ x_0 + \sum_{i \neq j}^n x_i &\geq D & \forall j = \{1 \dots n\} \\ x_i \in Z_+ & \forall i = \{0 \dots n\} \end{aligned}$$

where  $H$  and  $D$  are supposed integer and  $H \geq D$ . The  $Q_{n+1}(D, H)$  defining inequalities correspond in our model to the inequalities (21),(22),(29) and (30).

We are able to characterize completely the polyhedral structure of  $Q_{n+1}(D, H)$  so that the *cutting plane algorithm* applied to an optimization problem over  $Q_{n+1}(D, H)$  always yields the optimal integer solution. The  $F_n(L)$  polyhedral structure is, instead, more complex and future work will focus on this. In our testing on this polyhedron we have found an average gap between the MIP optimal value and its LP relaxation of 1,69% with a maximum of 4,25% and a minimum of 0,46%. Adding the *cuts* to the LP formulation the average difference between the MIP optimal value and the relaxed optimal value is of 0,18% with a maximum of 0,44% and a minimum of 0% (optimal).

Going back to the actual problem we want to solve, we have found that, after adding the “cuts” derived by our polyhedral study in [2], we have an average gap between the MIP optimal value and the relaxed optimal value of about 0,10% with a maximum of 0,37% and a minimum of 0 (i.e., optimal). If we apply the simple rounding heuristic however, i.e., setting to its ceiling each variable that is required to be integer, we obtain an average gap between the rounded solution value and the MIP value that is quite similar to the previous ones, i.e., 0,56% for the whole grid cost. We want to find a better heuristic.

In the sequel we will assume that we can increase and/or decrease by a very small amount each supplier's output (and hence reserve) without violating the transmission constraints. This "assumption" is not really such since it actually reflects the *Network Operator* modus operandi. In fact, when scheduling production, the *Network Operator* will never allow lines to be loaded up to their limit since it knows that, in real time operations, demand (and even supply) can diverge from the planned one and hence it needs to have room to real-time adjust production.

The heuristic we propose is the following. Let be  $P^* = (f_1^*, \dots, f_n^*, x_1^*, \dots, x_n^*, y_1^*, \dots, y_m^*)$  the optimal solution obtained from the cutting plane algorithm. If  $x_i$  and  $y_i$  are integer for all  $i$  then we have found the MIP solution. If that's not the case we note that the optimal MIP solution, let us call it  $\hat{P} = (\hat{f}_1, \dots, \hat{f}_n, \hat{x}_1, \dots, \hat{x}_n, \hat{y}_1, \dots, \hat{y}_m)$ , will have either  $f^* = \hat{f}$  or  $\sum_{i=1}^n f_i^* = \sum_{i=1}^n \hat{f}_i$  i.e.  $\hat{f}$  can be obtained from  $f^*$  by redistributing the flow. Suppose that we are in the first case, i.e.,  $f^* = \hat{f}$ . Note that this is going to be the case if energy costs are significantly higher than reservation costs. Hence, from constraint (23), it follows that  $\hat{x}_i \geq \lceil f_i^* \rceil$ . Also, note that, if it is feasible to decrease the flow by  $\epsilon$  on the  $x_i$  variable and increase by the same quantity a variable  $y_j$  then it must be the case that  $r_i \leq s_j$  since, otherwise, the point  $P^*$  wouldn't be optimal. Hence  $\hat{y}_j \leq \lceil y_j^* \rceil$ . Now, let be  $y_j$  such that  $s_j = \max\{s_i : m_i < y_j^*\}$  (where  $m_i$  is the variable lower bound) and let be  $q$  the number of positive components of  $x^*$  that are not integers. Let be  $z = \min(\lceil x_i \rceil - x_i)$  for all  $x_i$  not integer. Let be  $t = y_j - \lfloor y_j \rfloor \geq 0$  and  $u = \lfloor (q-1)z - t \rfloor$ . By setting  $w = \min(\lfloor y_j \rfloor - m_j, u)$  we can decrease  $y_j$  by  $w + t$  and increase each not integer variable  $x_i$  by  $\delta = \frac{t+w}{q-1}$  while remaining feasible. Assuming, without loss of generality, that the  $q$  not integer components in  $x^*$  are the first  $q$  we have that the point  $Q^a = (f_1^*, \dots, f_n^*, \lceil x_1^* + \delta \rceil, \dots, \lceil x_q^* + \delta \rceil, x_{q+1}^*, \dots, x_n^*, \lceil y_1^* \rceil, \dots, y_j^* - w - t, \dots, \lceil y_m^* \rceil)$  is feasible for the MIP and has a better objective value than the point  $R$  obtained by the simple rounding heuristic, namely,  $R = (f_1^*, \dots, f_n^*, \lceil x_1^* \rceil, \dots, \lceil x_n^* \rceil, \lceil y_1^* \rceil, \dots, \lceil y_m^* \rceil)$ . By our choice of  $\delta$ , in fact, is  $\lceil x_i^* + \delta \rceil = \lceil x_i^* \rceil$ . Moreover, if  $w = u$ , the point  $Q$  has, at least, two more coordinate integer, namely,  $y_j$  and  $x_l$  where  $z = \lceil x_l \rceil - x_l$ . If, instead,  $w < u$  then  $y_j$  will drop to its (integer) lower bound. Let us consider now  $k = \operatorname{argmax}\{r_i : x_i > \lceil f_i \rceil\}$ . If  $r_k > s_j$  then we need to check the value of another feasible point. Hence, let be  $z' = \min(\lceil x_i \rceil - x_i)$  for all  $x_i$  not integer and  $i \neq k$ . Let be  $t' = x_k - \lfloor x_k \rfloor \geq 0$  and  $u' = \lfloor (q-2)z' - t' \rfloor$ . By setting  $w' = \min(\lfloor x_j \rfloor - \lceil f_k \rceil, u')$  we can decrease  $x_k$  by  $w' + t'$  and increase each not integer variable  $x_i$ ,  $i \neq k$  by  $\delta' = \frac{t'+w'}{q-2}$  while remaining feasible. As before, one or two coordinates  $x_i$  will become integer. The point  $Q^b = ((f_1^*, \dots, f_n^*, \lceil x_1^* + \delta' \rceil, \dots, x_k^* - t' - w', \dots, \lceil x_q^* + \delta' \rceil, x_{q+1}^*, \dots, x_n^*, \lceil y_1^* \rceil, \dots, \lceil y_m^* \rceil)$  is feasible for the MIP and has a better objective value than the point  $R$  obtained by the simple rounding heuristic. We set  $Q^1 = \min(Q^a, Q^b)$  where the minimum is with respect the objective function and will reapply the procedure to the point  $P^1$  that is equal to the point  $Q^1$  except that we have removed all the ceiling from the not integer variables. At the end, we will get the point  $Q^t = (f_1^*, \dots, f_n^*, \lceil x_1^t \rceil, \dots, \lceil x_n^t \rceil, \lceil y_1^t \rceil, \dots, \lceil y_m^t \rceil)$  where  $t$  is the number of iteration of this procedure and where most of the  $x_i$  variable will be integer. Let us consider the point  $P^t = (f_1^*, \dots, f_n^*, \lceil x_1^t \rceil, \dots, \lceil x_n^t \rceil, y_1^t, \dots, y_m^t)$ . Let  $a = \operatorname{argmin}\{s_j : m_j \leq y_j^t < C_j\}$  and  $b = \operatorname{argmax}\{s_j : m_j < y_j \leq C_j\}$  and consider  $\delta = \min(\lceil y_a \rceil - y_a, y_b - \lfloor y_b \rfloor)$ . The point  $P^{t+1}$  obtained from  $P^t$  by increasing  $y_a$  by  $\delta$  and decreasing  $y_b$  by  $\delta$  is feasible, has a no worse objective value than  $Q^t$  and has at least one more component  $y_i$  integer. By reapplying the procedure, all the  $y$  components, except at most one, will be integer. By rounding this one component we will get a point  $Q^*$  feasible for the MIP and that will be our approximate solution for the MIP problem. The number of iterations,  $t$ , will be anyway small and most of the time equal to 1. In our testing this heuristic has always yield the optimal MIP solution when  $f^* = \hat{f}$ . If this is

not the case, though, the heuristic still provides a feasible point which is anyway better than the one obtained by simple rounding. In our testing, we came across one case in which  $f^* \neq \hat{f}$  and the heuristic provided a solution that was 0,02% away from the optimal MIP solution. We could easily improve the heuristic (by appropriately redistributing the flow among the existing positive variable plus one) to take care of the case in which  $f^* \neq \hat{f}$  but, since this is quite a rare event and the improvement is going to be probably quite small, we will not detail here the procedure.

The proposed scheme, the *cutting plane algorithm* coupled with the *heuristic*, can solve our optimization program very quickly and hence it can practically be used in energy markets.

## References

- [1] D. Armstrong, S. Cowan, and J. Vickers, “Regulatory reform: economic analysis and British experience”, *MIT Press*, Chapter 9, 1994.
- [2] D. Bienstock, G. Muratore, “Strong inequalities for capacitated survivable network design problems”, *Mathematical Programming*, vol. 89, 2000.
- [3] D. Bienstock, S. Mattia, “Using mixed-integer programming to solve power grid blackout problems”, *CORC Technical Report*, October 2005.
- [4] E. Bohn, C. Caramanis, C. Scheppe, “Optimal pricing in electrical networks over space and time”, *Rand Journal of Economics*, 15, 360–376, 1984.
- [5] F. Bolle, “Supply function equilibria and the danger of tacit collusion: the case of spot markets for electricity”, *Energy Economics*, 14, 94–102, 1992.
- [6] S. Borenstein, J. B. Bushnell, S. E. Stoft, “The competitive effects of transmission capacity in a deregulated electricity industry”, *Rand Journal of Economics*, 31, 294–325, 1998.
- [7] J. Boucher, Y. Smeers, “Alternative models of restructured electricity systems, part 1: No market power”, *Operations Research*, 49, 821–838, 2001.
- [8] G. M. Casolino, “Mercato elettrico liberalizzato: gestione ottimale con un approccio di decomposizione/coordinamento”, *PH.D. Thesis*, Universit degli Studi di Cassino, 2003.
- [9] H. Chao, H. G. Huntington, *Designing competitive electricity markets*, Kluwer Academic Press, 1998.
- [10] H. Chao, S. Peck, “An institutional design for an electricity contract market with central dispatch”, *Energy Journal*, 18, 85–110, 1997.
- [11] H. Chao, S. Peck, S. S. Oren, R. Wilson, “Flow based transmission rights and congestion management”, *The Electricity Journal*, 13, 38–58, 2000.
- [12] H. Chao, R. Wilson, “Design of wholesale electricity markets”, *Electric Power Research Institute*, 2001.
- [13] M. A. Einhorn, *From regulation to competition: new frontiers in electricity markets*, Kluwer Academic Publishers, 1994.
- [14] F. D. Galiana, M. Ilić, “A mathematical framework for the analysis and management of power transactions under open access”, *IEEE Transaction on Power Systems*, Vol. 13 No. 2, 681–687, 1998.
- [15] R. J. Green and D. M. Newbery, “Competition in the British electricity spot market”, *Journal of Political Economy*, 100(5), 929–953, 1992.
- [16] M. Grötschel, C. Monma and M. Stoer, “Design of survivable networks” *vol. Network Models of Handbooks in Operations research and Management Science*, chp. 10, (North-Holland), 617–672, 1995.

- [17] A. Henney, S. Crisp, “Lessons for the U.S.? Transmission pricing, constraints, and gaming in England and Wales”, *Electricity Journal*, 17-23, Jan-Feb 1997.
- [18] W. W. Hogan, “Contract networks for electric power transmission”, *J. Regulatory Econ.*, 211-242, 1992.
- [19] W. W. Hogan, “Market in real electric networks require reactive prices”, *The Energy Journal*, Vol. 14 No. 3, 1993.
- [20] W. W. Hogan, J. Chandley, S. Harvey, “Congestion management in California”, *Tech. Rep.*, John F. Kennedy School of Government, Harvard University, Cambridge, MA, 2000.
- [21] W. W. Hogan, “Successful market design and failure diagnosis: blackoutss and lamppost in regulating electricity markets”, *Tech. Rep.*, J.F.K School of Governement, Harvard University, USA, October 2003.
- [22] X. Hu, D. Ralph, E. K. Ralph, P. Bardsley, M. C. Ferris, “Electricity generation with looped transmission networks: Bidding to an ISO”, *preprint*, 2004.
- [23] M. Ili'c, E. Allen, Z. Younes, “Providing for transmission in times of scarcity: an ISO cannot do it all”, *International Journal of Electrical Power and Energy Systems*, Vol. 21, 1999.
- [24] P. Joskow, “Restructuring, competition and regulatory reform in the U.S. electricity sector”, *Journal of Economic Perspectives*, 11(3), 119–138, 1997.
- [25] P. Joskow and R. Schmalensee, *Markets for power*, MIT Press, 1983.
- [26] P. Joskow, J. Tirole, “Transmission rights and market power on electric power networks”, *Rand Journal*, 31, 450–487, 2000.
- [27] P. Neuhoff, “Integrating transmission and energy markets mitigates market power”, *Working Paper*, University of Cambridge, 2003.
- [28] H. Kerivin, A. R. Majhoub, “ On survivable network polyhedra”, *Discr. Math.*, (to appear).
- [29] G. Muratore, “Polyhedral approaches to survivable network design”, *Ph.D. Thesis*, Columbia University, New York, 1999.
- [30] K. Neuhoff, J. Barquin, M. G. Boots, A. Ehrenmann, B. F. Hobbs, F. A.M. Rijkers, “Network-constrained Cournot models of liberalized electricity markets: the devil is in the details”, *Energy Economics*, 2005.
- [31] D. Newbery, “Power markets and market power”, *Energy Journal*, 16(3), 41–66, 1995.
- [32] R. P. O'Neill, P. M. Sotkiewicz, B. F. Hobbs, M. H. Rothkopf, W. R. Stewart Jr, “Efficient market-clearing prices in markets with nonconvexities”, *European J. Operational Research*, 2004.
- [33] S. Oren, “Economic Inefficiency of passive transmission rights in congested electricity systems with competitive generation”, *Energy Journal*, 18(1), 63-84, 1997.
- [34] J. S. Pang, B. F. Hobbs, C. Day, “Electricity market models with conjectured resource price responses”, *Mathematical Programming B.*, vol.101 (1), 57-94, 2004.

- [35] D. Rajan , A. Atamturk, “A directed cycle-based column-and-cut generation method for capacitated survivable network design”, *Networks* 43 (4), 201–211, 2004.
- [36] R. Riley, “SeTrans market design issues”, *FERC SMD Technical Conference*, Atlanta GA, June 2003.
- [37] L. E. Ruff, “Competition in electricity: where do we go from here?”, *Lectures on Regulation Series VIII*, London Business School, October 1998.
- [38] F. C. Schweppe, M. Caramanis, R. Tabors, R. Bohn, *Spot pricing of electricity*, Kluwer Academic Publisher, Boston, 1988.
- [39] Y. Smeers, J. Y. Wei, “Spatial oligopolistic electricity models with Cournot generators and regulated transmission prices”, *Operations Research*, 47(1), 102–112, 1999.
- [40] M. Stoer, “Design of survivable networks ”, *Lecture Notes in Mathematics*, 1531, Springer-Verlag, Berlin, 1992.
- [41] J. Vickers and G. K. Yarrow, “The British electricity experiment”, *Economic Policy*, 12, 188–232, 1991.
- [42] N-H. Von der Fehr and D. Harbord, “Spot market competition in the U.K. electricity industry”, *Economic Journal*, 103, 531–546, 1993.
- [43] B. Willems, “Electricity networks and generation market power ”, *Ph.D. Thesis* , Faculteit Economische en Toegepaste Economische, Wetenschappen Katholieke Universiteit Leuven, 2004.
- [44] R. Wilson, “Architecture of power markets”, *Econometrica*, 70, 1299–1340, 2002.
- [45] F. Wolak, “Market design and price behavior in restructured electricity markets: an international comparison”, *Tech. Rep*, Energy Institute, University of California, Berkeley CA, 1997.
- [46] C. D. Wolfram, “Strategic bidding in a multi-unit auction: an empirical analysis of bids to supply electricity in England and Wales”, *Rand Journal of Economics*, 29, 703–725, 1998.
- [47] “White paper: wholesale power market platform”, *Federal Energy Regulatory Commission*, USA, April 2003.
- [48] “Interim report: causes of the August 14th blackout in the United States and Canada”, *Us-Canada Power System Outage task Force*, November 2003.
- [49] “Planning and operating standards under Beta”, *An OFGEM/DTI Conclusion Document*, 30 June 2003.
- [50] “Determinazione dell’equilibrio del mercato del giorno prima dell’energia con prezzo uniforme per i consumatori e prezzi zonali per i generatori”, *Analisi Tecnica n.5/02*, Gestore Mercato Elettrico, Ottobre 2002.