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A. Germani, C. Manes, P. Palumbo

**FILTERING OF SWITCHING SYSTEMS
VIA A SINGULAR MINIMAX APPROACH**

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Alfredo Germani – Dipartimento di Ingegneria Elettrica, Università degli Studi dell'Aquila, 67040 Monteluco (L'Aquila), Italy and Istituto di Analisi dei Sistemi ed Informatica del CNR, Viale Manzoni 30, 00185 Roma, Italy. Email: `germani@iasi.rm.cnr.it`.

Costanzo Manes – Dipartimento di Ingegneria Elettrica, Università degli Studi dell'Aquila, 67040 Monteluco (L'Aquila), Italy and Istituto di Analisi dei Sistemi ed Informatica del CNR, Viale Manzoni 30, 00185 Roma, Italy. Email: `manes@ing.univaq.it`.

Pasquale Palumbo – Istituto di Analisi dei Sistemi ed Informatica del CNR, Viale Manzoni 30, 00185 Roma, Italy. Email: `palumbo@iasi.rm.cnr.it`.

Abstract

This paper considers the problem of state estimation for discrete-time systems whose dynamics switches within a finite set of linear stochastic behaviors. In recent years such systems are receiving a growing attention because of their importance from an applicative point of view, in that switching phenomena are normally present in many engineering problems. The solution of the filtering problem depends on the amount of the *a priori* information about the switching process. In most papers the switching process is modeled by a discrete Markov chain, with a known transition matrix. For this problem the exact computation of the optimal filter is cumbersome, and most papers deal with the problem of computing approximate filters.

In this paper a switching process, that is not statistically characterized, is considered. The system is regarded as an uncertain regular system and it is transformed into a singular system with uncertainties only on the second order statistics of noises. This allows us to develop a minimax linear filter, that is the filter that gives the minimum error variance in the worst case of noise statistics.

Key words: State estimation, switching systems, hybrid systems, minimax filtering, singular systems.

1. Introduction

Switching systems, also denoted hybrid systems or variable structure systems, are receiving a growing attention in recent years because of their importance from an applicative point of view, in that switching phenomena are normally present in many engineering problems (for a survey on hybrid systems control and applications see [7,2,14]). Many authors investigated the problem of state estimation for switching systems. Most papers in literature deal with systems with a stochastically driven switching sequence, modeled as a finite-state Markov Chain (see e.g. [1, 3, 12, 9, 16, 8] for the discrete-time case and [13, 20, 19] for the continuous-time case).

This paper considers the case in which no statistical information is available on the switching process. We will show that in this case it is often still possible to estimate the state by considering the switching process as an additional state, whose dynamics is unknown, and by considering the noise processes with unknown and bounded second order statistics. One key tool to solve this problem is the theory of singular systems [6]. Such systems are characterized by a lack of information about some components of the dynamic equation [17]. In this paper, by using some new results on the filtering for stochastic singular systems [10] and following the minimax approach for filtering of uncertain stochastic linear systems [4, 15, 18] it will be shown how to overcome the lack of statistical informations.

2. Stochastic switching systems

We consider discrete-time stochastic systems whose dynamics switches between a finite number of linear functioning modes. Such systems can be described as follows:

$$
\begin{aligned}
x(0) &= x_0, \\
x(k+1) &= A_{\mu(k)}x(k) + B_{\mu(k)}u(k) + F_{\mu(k)}N_1(k), \quad k \geq 0, \\
y(k) &= C_{\mu(k)}x(k) + D_{\mu(k)}u(k) + G_{\mu(k)}N_2(k),
\end{aligned}
\tag{2.1}
$$

where $x(k) \in I\!\!R^n$ is the system state, $y(k) \in I\!\!R^q$ is the measured output and $u(k) \in I\!\!R^p$ is a known deterministic input. $N_1(k)$ and $N_2(k)$ are uncorrelated standard white noise sequences (identity covariance matrix), x_0 is a random vector with given mean $\bar{x}_0$ and covariance Ψ_{x_0}. The sequence $\{\mu(k)\}$ commands the switching between the finite modes.

Most papers in the literature model the sequence $\{\mu(k)\}$ as a finite-state Markov Chain. In this case system (2.1) is also called a *jump Markov linear system*. One of the first papers in the field is [1], in which only the switching of the noise covariance matrices $F_{\mu(k)}F_{\mu(k)}^T$ and $G_{\mu(k)}G_{\mu(k)}^T$ is considered. A widely appreciated approximate filtering approach is the one proposed in [3], while suboptimal fixed-interval smoothing was presented in [12]. In [9] exact filters are computed, whose complexity grows with time. Comparisons among different approximate state estimation algorithms are performed in [16] and in [8]. The case of continuous-time jump Markov linear systems has been considered in [13, 20, 19].

In this paper we study the filtering problem in the case of absence of any *a priori* statistical information on the switching sequence: nothing can be said on *when* a switch will occur and on *how long* it will last. With this assumption the switching system appears as a linear time-varying partially known system. In particular we consider the case of a binary unknown sequence $\mu(k) \in \{0, 1\}$. The dependence of the system matrices on the binary parameter $\mu(k)$ can be expressed either as

$$
A_{\mu(k)} = A_0\big(1 - \mu(k)\big) + \mu(k)A_1
\tag{2.2}
$$

4.

or as
$$A_{\mu(k)} = A_0 + \mu(k)A_\Delta, \qquad \text{with} \quad A_\Delta = A_1 - A_0. \tag{2.3}$$

The same can be said for matrices $B_{\mu(k)}, C_{\mu(k)}, D_{\mu(k)}, F_{\mu(k)}, G_{\mu(k)}$.

For our purposes, defining
$$\begin{aligned} A_\Delta &= A_1 - A_0, \quad B_\Delta = B_1 - B_0, \quad F_\Delta = F_1 - F_0, \\ C_\Delta &= C_1 - C_0, \quad D_\Delta = D_1 - D_0, \quad G_\Delta = G_1 - G_0, \end{aligned} \tag{2.4}$$

it is convenient to rearrange system (2.1) in the form

$$\begin{aligned} x(k+1) &= \big(A_0 + \mu(k)A_\Delta\big)x(k) + \big(B_0 + \mu(k)B_\Delta\big)u(k) + \big(F_0 + \mu(k)F_\Delta\big)N_1(k), \\ y(k) &= \big(C_0 + \mu(k)C_\Delta\big)x(k) + \big(D_0 + \mu(k)D_\Delta\big)u(k) + \big(G_0 + \mu(k)G_\Delta\big)N_2(k), \end{aligned} \tag{2.5}$$

Remark 2.1. In many practical situations only few coefficients of matrices $A_{\mu(k)}$ and $C_{\mu(k)}$ are subject to switching. An efficient parameterization of the switching effects can be made as follows. Let $s = \text{rank}\begin{bmatrix} A_1 - A_0 \\ C_1 - C_0 \end{bmatrix}$. Then a full row-rank matrix $S \in I\!R^{s \times n}$ can be found such that:
$$A_1 - A_0 = \widetilde{A}_\Delta S, \qquad C_1 - C_0 = \widetilde{C}_\Delta S, \tag{2.6}$$

with some $\widetilde{A}_\Delta$, $\widetilde{C}_\Delta$ in $I\!R^{n \times s}$ and $I\!R^{q \times s}$ respectively, with matrix $\begin{bmatrix} \widetilde{A}_\Delta \\ \widetilde{C}_\Delta \end{bmatrix}$ full column-rank.

3. Singular models for switching systems

System (2.5) is bilinear with respect to the pairs $\big(\mu(k), x(k)\big)$, $\big(\mu(k), u(k)\big)$, $\big(\mu(k), N_i(k)\big)$. No statistical knowledge is available for $\mu(k)$. Such situation is modeled by immerging the system (2.5) in a suitable stochastic singular system (see [6]). Taking account for the definition of matrix S given in Remark 2.1 a variable $z(k) \in I\!R^s$ can be defined as

$$z(k) = \mu(k)Sx(k) \tag{3.1}$$

and system (2.5) can be rewritten as

$$\begin{aligned} x(k+1) &= A_0 x(k) + \widetilde{A}_\Delta z(k) + B(k)\mu(k) + B_0 u(k) + F(k)N_1(k), \\ y(k) &= C_0 x(k) + \widetilde{C}_\Delta z(k) + D(k)\mu(k) + D_0 u(k) + G(k)N_2(k), \end{aligned} \tag{3.2}$$

where $B(k) = B_\Delta u(k)$, $D(k) = D_\Delta u(k)$, $F(k) = F_0 + \mu(k)F_\Delta$, $G_0 + \mu(k)G_\Delta$.

If we define an extended state $X(k) \in I\!R^{n+s+1}$ for the original system (2.5) as follows,

$$X(k) = \begin{bmatrix} x(k) \\ z(k) \\ \mu(k) \end{bmatrix} \in I\!R^{n+s+1}, \quad \text{with } z(k) \text{ given by (3.1)}, \tag{3.3}$$

the evolution of $X(k)$ can be described by a singular system, as stated by the following theorem:

Theorem 3.1. *Consider system (3.2) and define the extended state $X(k) \in I\!\!R^{n+s+1}$ as in (3.3). Then, the evolution of $X(k)$ is given by the following singular system:*

$$X(0) = [\, x^T(0) \quad \mu(0)x^T(0)S^T \quad \mu(0)\,]^T,$$
$$JX(k+1) = A(k)X(k) + B_0u(k) + f(k), \qquad k \geq 0, \tag{3.4}$$
$$y(k) = C(k)X(k) + D_0u(k) + g(k),$$

with

$$J = \begin{bmatrix} I_n & O_{n\times s+1} \end{bmatrix}, \qquad \begin{aligned} A(k) &= \begin{bmatrix} A_0 & \widetilde{A}_\Delta & B_\Delta u(k) \end{bmatrix}, \\ C(k) &= \begin{bmatrix} C_0 & \widetilde{C}_\Delta & D_\Delta u(k) \end{bmatrix}, \end{aligned} \tag{3.5}$$

and with uncorrelated white sequences $\{f(k)\}$ and $\{g(k)\}$ defined as $f(k) = F(k)N_1(k)$ and $g(k) = G(k)N_2(k)$, whose covariance matrices are

$$Q(k) = E\{f(k)f^T(k)\} = F_0 F_0^T \left(1 - \mu(k)\right) + F_1 F_1^T \mu(k),$$
$$R(k) = E\{g(k)g^T(k)\} = G_0 G_0^T \left(1 - \mu(k)\right) + G_1 G_1^T \mu(k). \tag{3.6}$$

The mean and covariance of the initial descriptor vector $X(0)$ are:

$$I\!\!E\{X(0)\} = \begin{bmatrix} \bar{x}_0 \\ \mu(0)S\bar{x}_0 \\ \mu(0) \end{bmatrix}, \qquad \Psi_{X(0)} = \begin{bmatrix} \Psi_{x_0} & \mu(0)\Psi_{x_0}S^T & O_{n\times 1} \\ \mu(0)S\Psi_{x_0} & \mu(0)S\Psi_{x_0}S^T & O_{n\times 1} \\ O_{1\times n} & O_{1\times n} & 0 \end{bmatrix}.$$

Proof. Equivalence of systems (3.4) and (2.5) is obtained by simple substitution in (3.4) of the definitions of J, $A(k)$, $C(k)$, $X(k)$ and with the substitutions $f(k) = F(k)N_1(k)$ and $g(k) = G(k)N_2(k)$. The covariances (3.6) of the state noise $F(k)N_1(k) = (F_0(1 - \mu(k)) + \mu(k)F_1)N_1(k)$ and of the observation noise $G(k)N_2(k) = (G_0(1 - \mu(k)) + \mu(k)G_1)N_2(k)$ can be easily computed recalling that $\mu(k)$ is not a random sequence. Also the mean and covariance of $X(0)$ can be easily computed. ∎

In the case in which in the original system (2.5) we have $B_0 = B_1$ and $D_0 = D_1$, so that both matrices B_Δ and D_Δ vanish, it is more useful to define then a reduced descriptor vector as

$$\overline{X}(k) = \begin{bmatrix} x(k) \\ z(k) \end{bmatrix} \in I\!\!R^{n+s}, \quad \text{with } z(k) \text{ given by (3.1).} \tag{3.7}$$

The dynamics of $\overline{X}(k)$ is given by the following theorem:

Theorem 3.2. *Consider system (3.2) and define the extended state $\overline{X}(k) \in I\!\!R^{n+s}$ as in (3.7). Then, the evolution of $\overline{X}(k)$ is given by the following singular equation:*

$$\overline{X}(0) = [\, x^T(0 \quad \mu(0)x^T(0)S^T \,]^T,$$
$$\overline{J}\overline{X}(k+1) = \overline{A}\,\overline{X}(k) + B_0u(k) + f(k), \qquad k \geq 0, \tag{3.8}$$
$$y(k) = \overline{C}\,\overline{X}(k) + D_0u(k) + g(k),$$

with

$$\overline{J} = \begin{bmatrix} I_n & O_{n\times s} \end{bmatrix}, \qquad \begin{aligned} \overline{A} &= \begin{bmatrix} A_0 & \widetilde{A}_\Delta \end{bmatrix}, \\ \overline{C} &= \begin{bmatrix} C_0 & \widetilde{C}_\Delta \end{bmatrix}, \end{aligned} \tag{3.9}$$

and where the state and output noise sequences are as in Theorem (3.1). The mean and covariance of the initial descriptor vector $X(0)$ are:

$$\mathbb{E}\{\overline{X}(0)\} = \begin{bmatrix} \bar{x}_0 \\ S\bar{x}_0 \end{bmatrix}, \quad \Psi_{\overline{X}(0)} = \begin{bmatrix} \Psi_{x_0} & \mu(0)\Psi_{x_0}S^T \\ \mu(0)S\Psi_{x_0} & \mu(0)S\Psi_{x_0}S^T \end{bmatrix}. \tag{3.10}$$

Proof. Similar to the one of Theorem (3.1).

∎

At this point the original switching system has been transformed into a singular system, so that we are in a position to employ estimation techniques developed for singular systems. In particular, we are going to apply the approach described in [10]. First, we need to recall that in the state estimation problem for singular systems of the form (3.4) or (3.8) it is important to consider the following matrices (see [5])

$$H(k) = \begin{bmatrix} J \\ C(k) \end{bmatrix} = \begin{bmatrix} I_n & O_{n\times s} & 0_{n\times 1} \\ C_0 & \widetilde{C}_\Delta & D_\Delta u(k) \end{bmatrix}, \quad \text{for system (3.4)}$$

$$\overline{H} = \begin{bmatrix} \overline{J} \\ \overline{C} \end{bmatrix} = \begin{bmatrix} I_n & O_{n\times s} \\ C_0 & \widetilde{C}_\Delta \end{bmatrix}, \quad \text{for system (3.8)}$$

(3.11)

In particular, the singular system (3.4) is said to be *estimable from the measurements* if $\text{rank}(H(k)) = n + s + 1$, $\forall k \geq 0$, while the singular system (3.8) is *estimable from the measurements* if $\text{rank}(\overline{H}) = n + s$ (see [5]).

Remark 3.3. It is known that singular systems forced by a given input sequence admit, in general, infinite state trajectories for each feasible initial state. A singular system that is *estimable from the measurement* is such that only one trajectory starting from a given initial state is compatible with a measured output. In other words, the knowledge of the input, of the output and of the initial state univocally determines the system trajectory. In particular, the computation of the trajectory can be obtained by the evolution of a regular system that can be associated to the singular one, called a *Complete Regular System* (CRS) associated to the singular system (see [10]).

From what discussed in the previous Remark and from the structure of matrix $H(k)$ defined in (3.11), it can be understood that the parameter $\mu(k)$ can be estimated if the matrices B_Δ and D_Δ are not zero, together with some *persistent excitation assumption* on $u(k)$. The simplest assumption is that $u(k) \neq 0$ for all $k \geq 0$.

Theorem 3.4. *Consider system (2.5) with $B_\Delta = 0$ and $D_\Delta = 0$, and such that the matrix $\overline{H}$ defined in (3.11) has rank $s + n$. Then, the following Complete Regular System describes the evolution of $\overline{X}(k)$:*

$$\overline{X}(k+1) = \overline{\mathcal{A}}X(k) + \overline{\mathcal{D}}y(k+1) + \mathcal{F}(k),$$
$$y(k) = \overline{C}X(k) + \mathcal{G}(k) \tag{3.12}$$

with initial state as in (3.8) and with:

$$\overline{\mathcal{A}} = \overline{H}^+ \begin{bmatrix} \overline{A} \\ O_{q\times(n+s)} \end{bmatrix}, \quad \overline{\mathcal{D}} = \overline{H}^+ \begin{bmatrix} O_{n\times q} \\ I_q \end{bmatrix}, \quad \mathcal{F}(k) = \overline{H}^+ \begin{bmatrix} f_k \\ -g_{k+1} \end{bmatrix}, \quad \mathcal{G}_k = g_k \tag{3.13}$$

where $\overline{H}^+$ denotes any left-inverse of $\overline{H}$.

Proof. By assumption the singular system (3.8) is estimable from the measurements $(\text{rank}(\overline{H}) = n + s)$, so that, following the theory developed in [10], a Complete Regular System of the form (3.12) can be associated to it.

■

Remark 3.5. The construction of a CRS is the basis for the construction of a filter for a singular system that is estimable from the measurement. Under the assumption of $\text{rank}(H(k)) = n + s + 1$ we can associate a CRS also to the singular system (3.4). However, for the sake of brevity, in the following only the solution of the filtering problem for the system (3.8) will be developed.

4. Minimax filtering for switching systems

A switching system of the form (2.1), with an unknown switching sequence $\{\mu(k)\}$ that is not stochastically modeled, can be regarded as an uncertain system, with uncertainties in all system matrices. In the previous section we defined uncertain singular systems (3.4) and (3.8) with uncertainties only on the initial state mean vector and covariance matrix, and on the covariances of the noise sequences. The values of the covariances at a given instant k depend on the value assumed by the switching parameter $\mu(k)$.

An appealing approach to deal with stochastic estimation problems with partial knowledge of the involved statistics, is the so-called minimax criterion [4, 15, 18]. The minimax approach consists in finding the best estimator in the worst case with respect to the class of admissible estimators. In the case in which the noise and the initial state covariances are unknown and bounded, with known bounds, in [4] it has been shown that the minimax filter is the Kalman filter with the Riccati equations forced by the known upper bounds on the covariances. In order to find an upper bound for the initial state covariance matrix, in the following we will assume that the initial state of the switching system (2.1) is a zero mean random variable, i.e. $\bar{x}_0 = 0$ so that $\mathbb{E}\{\overline{X}(0)\} = 0$.

Now we can define matrices $\overline{\Psi}$, $\overline{Q}$ and $\overline{R}$ such that, for any sequence $\{\mu(k)\}$

$$\Psi_{\overline{X}(0)} \leq \overline{\Psi}, \quad Q(k) \leq \overline{Q}, \quad R(k) \leq \overline{R}, \tag{4.1}$$

where $\Psi_{\overline{X}(0)}$ is defined in (3.10) and $Q(k)$, $R(k)$ are defined in (3.6).

Moreover we need to consider the following decomposition of the descriptor vector of system (3.8):

$$\overline{X}(k) = X^{nc}(k) + X^c(k) \tag{4.2}$$

where $X^{nc}(k)$ satisfies the equation:

$$X^{nc}(k + 1) = \overline{\mathcal{A}}X^{nc}(k) + \overline{\mathcal{D}}y(k + 1), \qquad X^{nc}(0) = 0, \tag{4.3}$$

not directly affected by the noise, evolving as a deterministic linear system forced by the measurements (non strictly causal system). The component $X^c(k)$ satisfies the equation:

$$\begin{aligned} X^c(k + 1) &= \overline{\mathcal{A}}X(k)^c + \mathcal{F}(k), \qquad X^c(k_0) = \overline{X}(0) \\ Y^c(k) &= y(k) - \overline{C}X^{nc}(k) = \overline{C}X^c(k) + \mathcal{G}(k) \end{aligned} \tag{4.4}$$

and evolves as a stochastic linear system.

8.

Definition 4.1. Consider the vector $\mathcal{Y}_k^c$ of all measurements Y^c up to time k, defined as

$$\mathcal{Y}_k^c = \begin{bmatrix} Y^c(0) \\ \vdots \\ Y^c(k) \end{bmatrix} \in I\!R^{q(k+1)}, \tag{4.5}$$

and let Π denote any real $(n+s) \times q(k+1)$ matrix. Then the minimax state estimation for system (2.1) is defined by:

$$\hat{x}(k) = \begin{bmatrix} I_n & O_{n \times d} \end{bmatrix} (X^{nc}(k) + \widehat{X}^c(k)) \tag{4.6}$$

where $\widehat{X}^c(k) = \widehat{\Pi}\mathcal{Y}_k^c$ is the solution of the minimax problem [4]:

$$\min_{\Pi \in I\!R^{(n+d) \times q(k+1)}} \quad \max_{\substack{Q(j) \leq \overline{Q} \\ R(j) \leq \overline{R} \\ \Psi_{\overline{X}(0)} \leq \overline{\Psi}}} \quad I\!E \left\| X^c(k) - \Pi\mathcal{Y}_k^c \right\|^2, \tag{4.7}$$

Remark 4.2. Definition 4.1 is based on some aspects of the decomposition (4.2), already discussed in [10]: a minimax estimate for the extended state $\overline{X}(k)$ has to be intended as the sum of the component $X^{nc}(k)$, directly available as a linear transformation of the measurements up to the current time k, and a minimax estimate for the component $X^c(k)$, filtered from the measurements up to time k. It has to be stressed that $X^{nc}(k)$, due to its structure, is itself best estimate among all the Borel functions of the measurements $\mathcal{Y}_k^c$ and, moreover, no uncertainty affects its computation.

Remark 4.3. Note that the error covariance matrix of $\hat{x}(k)$ is such that

$$\mathrm{Cov}\big(x(k) - \hat{x}(k)\big) = \begin{bmatrix} I_n & O_{n \times s} \end{bmatrix} \mathrm{Cov}\big(X^c(k) - \widehat{X}^c(k)\big) \begin{bmatrix} I_n \\ O_{s \times n} \end{bmatrix}, \tag{4.8}$$

and therefore it depends only on the component $X^c(k)$ of the extended state.

The structure of the minimax filter for system (2.1), taking into account all definitions of matrices $\overline{\mathcal{A}}$, $\overline{\mathcal{D}}$, $\overline{C}$, $\overline{Q}$, $\overline{R}$, $\overline{\Psi}$, is given by the following theorem.

Theorem 4.4. *The linear minimax state estimate for the switching system (2.1) is given by the equations:*

$$\widehat{\overline{X}}(k+1) = \overline{\mathcal{A}}\widehat{\overline{X}}(k) + \overline{\mathcal{D}}y(k+1) + K(k+1)\Big(y(k+1) - \overline{C}\overline{\mathcal{D}}y(k+1) - \overline{C}\overline{\mathcal{A}}\widehat{\overline{X}}(k)\Big)$$

$$\widehat{\overline{X}}(0) = K(0)y(0) \tag{4.9}$$

$$\hat{x}(k) = \begin{bmatrix} I_n & O_{n \times d} \end{bmatrix}\widehat{\overline{X}}(k+1)$$

with the filter gain given by the Riccati equations:

$$M_P(k+1) = \begin{bmatrix} \overline{A}P(k)\overline{A}^T + \overline{Q} & O_{n \times q} \\ O_{q \times n} & \overline{R} \end{bmatrix}$$

$$K(k+1) = -\overline{H}^+ M_P(k+1)L^T \left(LM_P(k+1)L^T \right)^\dagger$$

$$P(k+1) = \left(\overline{H}^+ + K(k+1)L \right) M_P(k+1)\overline{H}^{+T}$$

$$P_P(k+1) = \overline{H}^+ M_P(k+1)\overline{H}^{+T} \tag{4.10}$$

$$P_P(0) = \overline{\Psi}, \qquad P(0) = \left(I_{n+s} - K(0)\overline{C} \right)\overline{\Psi}$$

$$K(0) = \overline{\Psi}\overline{C}^T \left(\overline{C}\overline{\Psi}\overline{C}^T + \overline{R} \right)^\dagger$$

with $L = \begin{bmatrix} O_{q \times n} & I_q \end{bmatrix} \cdot \left(I_{n+q} - \overline{H}\overline{H}^+ \right).$

Proof. The filter can be constructed following the same procedure developed in [10]) for the problem of linear filtering of a descriptor system. In the present case, however, instead of the classical Kalman filtering is used the minimax algorithm developed by [4], that consists to write down the Kalman filter equations using the upper bounds for the noise covariance matrices. ∎

5. Simulation results

In order to test the theory developed in this paper, numerical simulations have been produced. Without loss of generality, according to Remark 3.5, the system to be filtered has been supposed as the one described in (3.8) with no deterministic drifts. Numerical data are the following:

$$A_0 = \begin{bmatrix} 0.1 & 0 & 1 \\ 0 & 0.8 & -1 \\ 0.3 & 0 & 0.1 \end{bmatrix}, \quad C_0 = \begin{bmatrix} 0.5 & 1 & 0 \\ 0 & 1 & -2 \end{bmatrix}, \quad F_0 = \begin{bmatrix} 1 \\ -0.6 \\ 1 \end{bmatrix}, \quad G_0 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0.8 & 0 & 1 \\ 0 & 0.8 & -1 \\ -0.3 & 0 & 0.1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} -0.5 & 1 & 0 \\ 0 & 1 & -2 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 0.1 \\ -0.5 \\ -1.3 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 1.2 \\ -0.5 \end{bmatrix}$$

$$\tag{4.11}$$

In order to show the efficiency of the algorithm, the state and output noise sequences $N_i(k)$, $i = 1, 2$ are zero-mean standard sequences, whose distribution is the following:

$$P(N_1(k) = -2) = 0.2, \qquad P\left(N_2(k) = -\frac{\sqrt{6}}{3} \right) = 0.6,$$

$$P\left(N_1(k) = \frac{1}{2} \right) = 0.8, \qquad P\left(N_2(k) = \frac{\sqrt{6}}{2} \right) = 0.4, \tag{4.12}$$

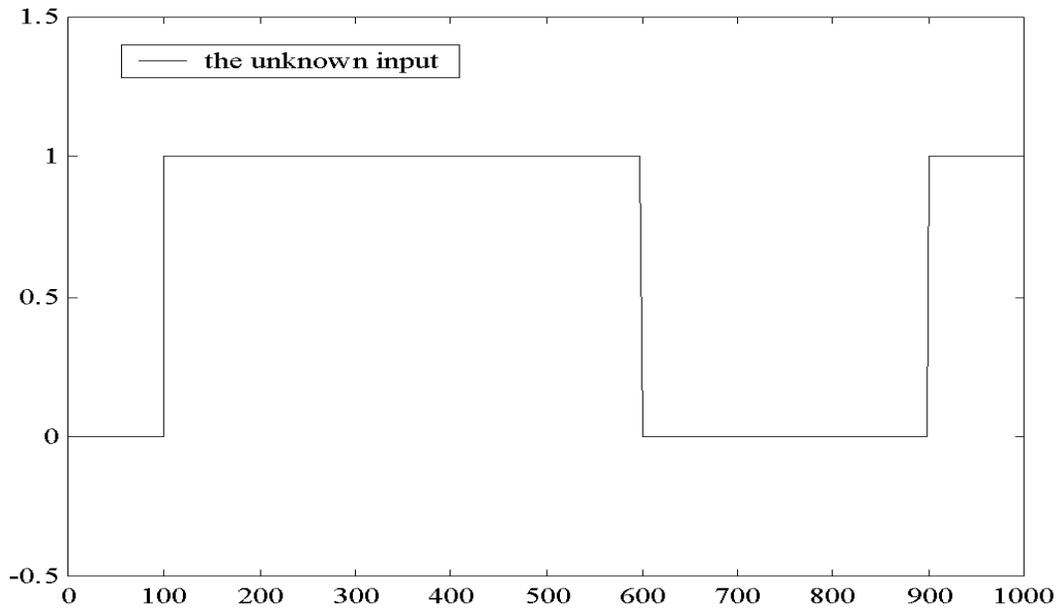The switch $\mu(k)$ is given the following sequence:

10.

Fig. 5.1 – The binary unknown sequence $\mu(k)$

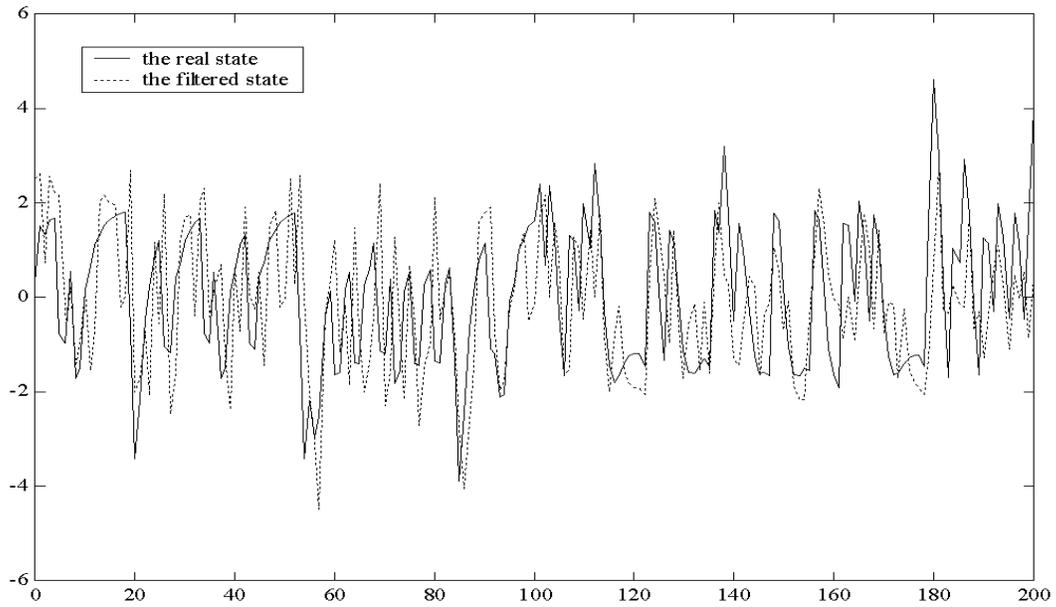The pictures below report the filtered state compared with the real one.

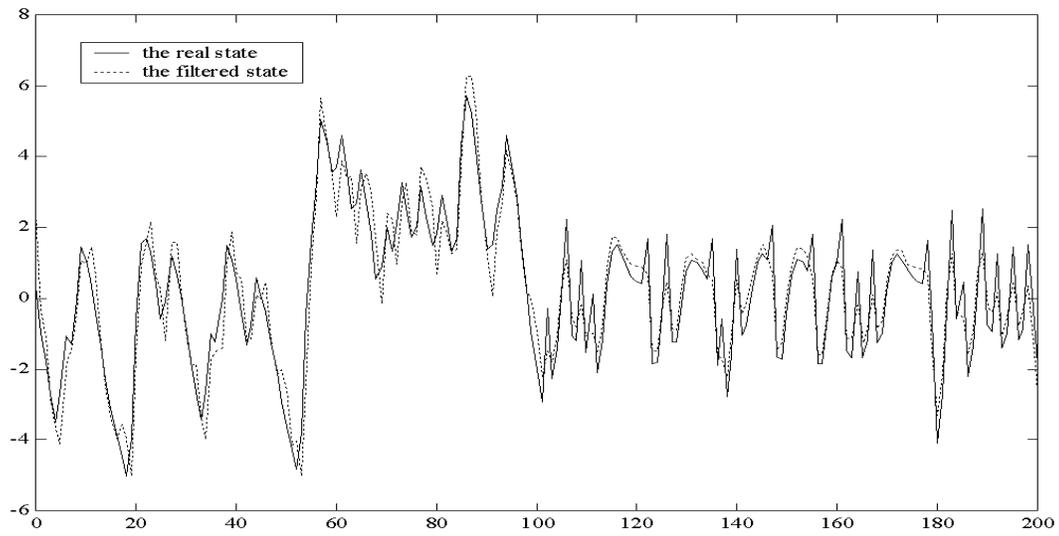Fig. 5.2 – True and estimated state: the first component.

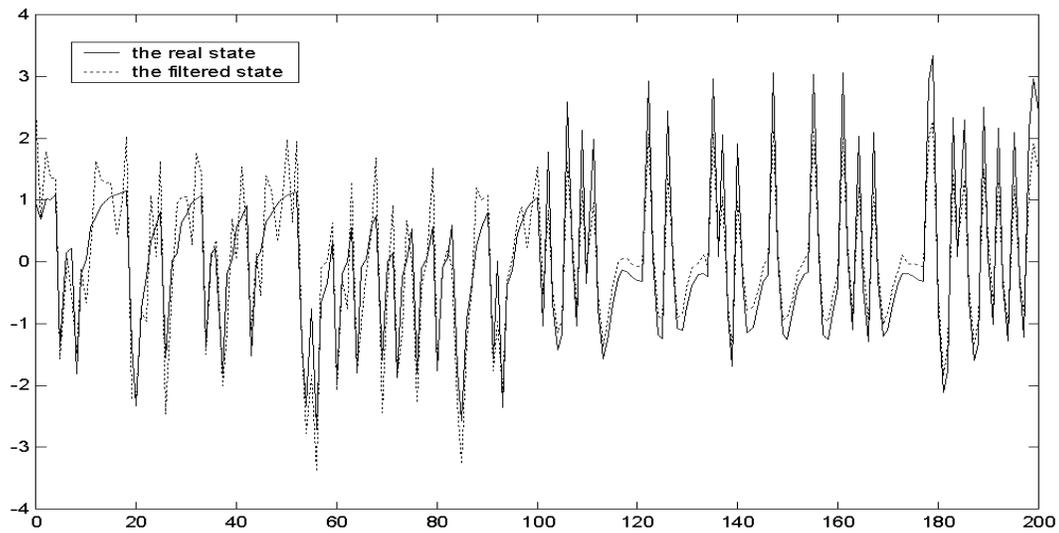Fig. 5.3 – True and estimated state: the second component.

Fig. 5.4 – True and estimated state: the third component.

12.

6. Conclusions

This paper proposes a filtering algorithm for the state estimate of an important class of discrete-time uncertain systems whose dynamics switches between a number of finite linear stochastic behaviors. No statistical *a priori* information is supposed available on the switching parameter. The approach here followed is taken from the theory of singular systems [6], which are systems characterized by a lack of information about some components of the dynamic equation [17]. By using some new results on the filtering for stochastic singular systems [10] and following the minimax approach for filtering of uncertain stochastic linear systems [4, 15, 18] a minimax linear filter is achieved, that is the filter that gives the minimum error variance in the worst case noise statistics. Numerical simulation show the goodness of the proposed filter, also in presence of highly asymmetric noise distributions.

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