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# COMPACT LOW FIELD MRI MAGNET: DESIGN AND OPTIMIZATION

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## Abstract

Magnetic Resonance Imaging (MRI) is performed with a very large instrument that allows the patient to be inserted into a region of uniform magnetic field. The field is generated either by an electromagnet (resistive or superconductive) or by a permanent magnet. Electromagnets are designed as air cored solenoids of cylindrical symmetry, with an inner bore of 80-100 cm in diameter. In clinical analysis of peripheral regions of the body (legs, arms, foot, knee etc.) it would be better to adopt much less expensive magnets leaving the most expensive instruments to applications that require the insertion of the patient in the magnet (head, thorax, abdomen, etc.). These "dedicated" apparati could be smaller and based on resistive magnets that are manufactured and operated at very low cost particularly if they utilise an iron yoke to reduce power requirements. In order to obtain a good field uniformity without the use of a set of shimming coils, we propose both a particular construction of a dedicated magnet, using four independently controlled pairs of coils, and an optimization-based strategy for computing, *a posteriori*, the optimal current values. The optimization phase could be viewed as a low-cost shimming procedure for obtaining the desidered magnetic field configuration. Some experimental measurements, confirming the effectiveness of the proposed approach, have also been reported. In particular, we show that the adoption of the proposed optimization-based strategy has allowed the achievement of a good uniformity of the magnetic field in about one fourth of the magnet length and about half of its bore. On the basis of the experimental results, the magnet can be successfully used in MRI applications.

## 1. Introduction

Magnetic Resonance Imaging (MRI) is one of the most useful tools for clinical diagnosis and biomedical research. It is performed with a very large instrument that allows the patient to be inserted into a region of uniform magnetic field. The field is generated either by an electromagnet (resistive or superconductive) or by a permanent magnet. Its intensity depends on the technology used and ranges from 0.02 T to 1.5 T ([1], [2]), values utilised in recently introduced commercial equipment. The magnet represents the most expensive part of the equipment. Superconductive or resistive magnets are designed as air cored solenoids of cylindrical symmetry, with an inner bore of 80-100 cm in diameter. The field in this case is directed along the axis of the cylinder.

Theoretically, the design of air cored solenoidal magnets is based on the principle that a current, produced by a coil built on the surface of a sphere with a surface current density that follows a cosine distribution, will produce a uniform magnetic field in the whole volume of the sphere [3]. Flat coils of diameter $2a$ and separation $a$, named Helmholtz coils, are a very crude approximation of a spherical magnet. A more accurate design is obtained by two or more pairs of coils in which spacing, diameter and thickness are evaluated in such a way as to achieve a cancellation of high order terms in the development of the field in a series of powers. Use of an iron enclosure makes accurate calculation of the field unfeasible due to the nonlinear characteristic of iron permeability. For this reason, MRI magnets are built without an iron yoke and generate a field distribution extending in all directions at distances of the order of a few times the magnet size, affecting ancillary electronic apparatus and being affected by the presence of metal structures. Another consequence of this construction is that it requires a large amount of power that must be dissipated by a cooling system. The weight of these magnets is about 15,000 kg.

In clinical analysis of peripheral regions of the body (legs, arms, foot, knee etc.) it would be better to adopt much less expensive magnets leaving the most expensive instruments to applications that require the insertion of the patient in the magnet (head, thorax, abdomen etc.). These "dedicated" apparati could be smaller and based on resistive magnets that are manufactured and operated at very low cost, particularly if they utilise an iron yoke to reduce power requirements. However, the reduced magnet size makes it difficult to obtain a uniform magnetic field in a large part of the magnet volume. Moreover, the lack of a rigorous characterization of the commercial iron used for the screen prevents the achievement in practice of the field uniformity predicted by field simulation programs.

In order to obtain a good field uniformity without the use of a set of shimming coils, we propose both a particular construction of a dedicated magnet, using four independently controlled pairs of coils, and an optimization-based strategy for computing, *a posteriori*, the optimal current values. The optimization phase could be viewed as a shimming procedure for obtaining the desidered magnetic field configuration. The experimental measurements confirm the effectiveness of the proposed approach (construction and optimization). Therefore, the dedicated magnet can be used for MRI of peripheral regions of the body and for animal experimentation at very low cost.

## 2. Magnet description and construction

The cross section of the cylindrical magnet and the main dimensions are shown in Figure 1. The magnet field is generated by the four couples of symmetric coils labelled 1 to 4. They are laid on an internal copper support which is part of the gradient assembly. The coils are wound using copper tape 6.3 mm wide and 2mm in thick in order to achieve good thermal conductivity

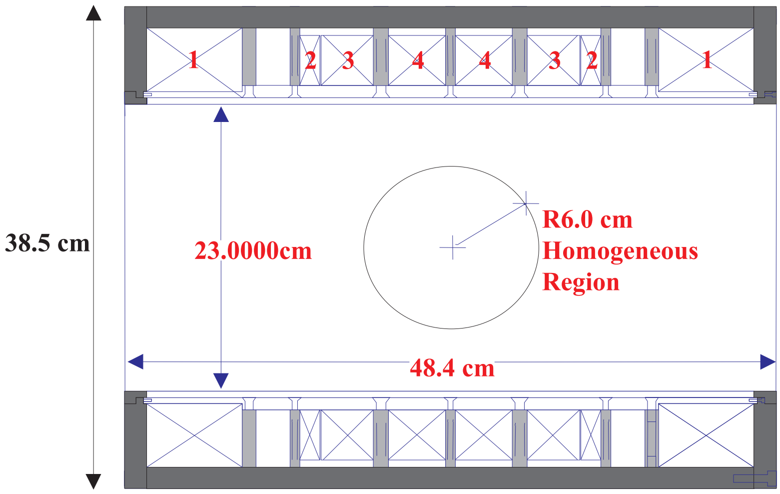


Figure 1: Longitudinal cross section of the magnet. Numbered from 1 to 4 are the pairs of symmetric coils that generate the main magnetic field. The external iron structure is indicated in dark gray. The circle represents the homogeneous region. The aluminium disks are indicated in lighter grey.

and to optimize the space factor. The energising currents are provided by 4 remotely controlled power supplies of nominal short term stability of 20 part per million, controlled by four, 16-bit, digital to analog converters (DAC). The magnet is presently cooled by a raw water, in future a closed circuit system using deionised water will be adopted.

The surrounding iron structure has the function of

a) confining the magnetic flux to avoid interference with the surrounding metal structure;

b) providing a low resistance path for the magnetic field flux;

c) contributing to the end effects correction.

It is built of thermally treated low carbon steel to improve the magnetic properties. Two removable end rings allow the field gradients assembly to be inserted or removed.

To avoid the problems related to the iron hysteresis cycle, the power supply of the two larger coils provides a 50 Hz amplitude controlled current, as used in demagnetisers. This reduces virtually to zero the residual iron magnetisation. In this way, when the magnet is switched on, the iron magnetisation always moves along the same B-H curve.

The gradients assembly consists of three set of coils to produce magnetic field gradients along the three orthogonal spatial directions and are used to encode three dimensional information. They have been designed by standard computation techniques [4], [5], [6] and built as printed circuit board using 1 mm thick copper sheet. The gradient coils are embedded in a cylindrical fiberglass structure (20 cm in internal diameter) to achieve the required mechanical stability. This structure is fixed to the internal copper tube, which has the function of screening the stationary field coils from the gradient coils. The tube (25 cm in external diameter, 48 cm in length) has a thickness of 1 cm in the central region and reduces to 0.6 cm in correspondence with the larger coils. Seven aluminium disks are fixed to the central tube and to the external iron cylinder. They limit the regions in which the coils are wound and keep the central copper tube in position. The presence of a tick copper screen is useful for avoiding the effects of transients in the magnet power supplies and for the accuracy of gradient computation.

## 3. Coils construction

The coils, generating the main magnetic field, have been built using copper tape in a pancake type construction that made it possible to obtain a space factor of about 0.9 and a thermal conductivity $\lambda_c$ of about $10^{-2}$ W/$^0$C cm. The copper coils are tightly wound on the internal copper cylinder and kept in position by the aluminium rings. The heat is removed by water cooling the central copper cylinder to maintain a maximum temperature rise of about 40$^0$C. The positions, dimensions and intensities of the coil currents have been selected on the basis of simulations by a standard of static field computation [7].

Field uniformity could, in principle, be optimized by building a single coil and optimizing the profile of this coil. This technique would have the advantage of requiring a single current to energise the magnet. Unfortunately, the nonlinearity of iron permeability and the limited precision of the mechanical construction would have required the use of shimming coils to achieve the required field uniformity.

The four symmetrical sets of coils and the four different power supplies have the same function as shimming coils and allow the achievement of a good field uniformity. From a point of view of the design and construction of the magnet, the use of four high current power supplies is only apparently more difficult to implement than the usual technique (based on a main coil with the addition of shimming coils). The difficulties are in any case related to the use of stable current sensors and to a low noise design.

The use of four power supplies makes the assumption that the field id symmetrical around the central cross section of the magnet. Due to the nonlinearity of iron permeability and to the limited tolerance of the mechanical construction, the measured magnetic field is very different from the numerical calculated value, both in field value and uniformity. This fact necessitates the use of a further optimization process to improve the quality of the magnetic field in terms of uniformity.

Having fixed the positions and the dimensions of the coils, the only parameters that can be changed are the intensities of the currents. For this reason, a set of four pairs of independent currents has been used. Another important parameter is the power required to achieve a given field intensity. This parameter is important in a compact construction in which power dissipation can become a serious design problem. In the present case, the total power required to achieve a field of 0.1T is about 2.2 kWatt.

## 4. Field optimization

As previously mentioned, the field is generated by four symmetric pairs of coils, say $I_1$, $I_2$, $I_3$, $I_4$. The required field value, say $B_0$, is 0.1T and it must be uniform as possible in a spherical region $\Omega$, 12 cm in diameter at the centre of the magnet. The vector of currents is denoted by $I = (I_1, I_2, I_3, I_4)$, and the magnetic field generated in a point $r$, which is directed along the magnet axis $z$, by $B_z(I, r)$. The problem that we want to solve through the optimization is to find a vector of currents $I^*$ such that

$$\left| \frac{B_z(I^*, r) - B_0}{B_0} \right| \leq \epsilon \qquad \text{for all } r \in \Omega,$$

where $\epsilon$ is a sufficiently small positive number. In the present case, the required maximum field deviation was $5\mu$T and $\epsilon$ has been taken equal to about $50 \times 10^{-6}$. The function $B_z : R^4 \to R$ defined in $\Omega$ is not known analytically, but for each $I$ and $r \in \Omega$ the the values $B_z(I, r)$ can be obtained experimentally. For this reason, we propose the following optimization strategy:

- determine a sufficient finite number $N_p$ of uniformly distributed points $r_j \in \Omega$, namely points such that, if for some $I^0 \in R^4$ we have

$$\left| \frac{B_z(I^0, r_j) - B_0}{B_0} \right| \leq \epsilon \qquad \text{for } j = 1, \ldots, N_p, \tag{1}$$

  then we can assume

$$\left| \frac{B_z(I^0, r) - B_0}{B_0} \right| \leq \epsilon \qquad \text{for all } r \in \Omega; \tag{2}$$

- for each point $r_j$, with $j \in \{1, \ldots, N_p\}$, define an analytical expression $\tilde{B}_z(I, r_j)$ approximating the true magnetic field $B_z(I, r_j)$;
- solve the unconstrained optimization problem

$$\min_{I \in R^4} \quad \sum_{j=1}^{N_p} (\tilde{B}_z(I, r_j) - B_0)^2. \tag{3}$$

We note that problem (3) is a nonlinear least squares problem, whose objective function is known analytically. The generic term $(\tilde{B}_z(I, r_j) - B_0)^2$ measures the difference between an estimate $\tilde{B}_z(I, r_j)$ of the magnetic field generated by a vector of currents $I$ in a point $r_j$ and the desidered value $B_0$. An optimization method using derivative information of any order can be used to solve (3) ([8], [9]). In our computation, we have employed the routine E04DGF of the NAG library which implements a quasi-Newton method.

Now, let $I^0$ be a global solution of (3). If the quantities $|\tilde{B}_z(I^0, r_j) - B_0|/B_0$, for $j = 1, \ldots, N_p$, are “sufficiently small” and the degree of accuracy of the models $\tilde{B}_z(I, r_j)$ is “high”, then, reasonably, condition (1) holds. Therefore, if the number of points $N_p$ is sufficiently large and the points are well distributed, also condition (2) is satisfied and $I^0$ represents the desidered vector of currents $I^*$.

On this basis, assuming that the points $r_j$ are well distributed (in the sense that (1) implies (2)), we have that the effectiveness of the proposed strategy depends from the reliability of the analytical model of the true magnetic field used to define the objective function in problem (3). For this reason, it is required an accurate representation of the magnetic field as function of the four currents.

## 5. The analytical model of the magnetic field

At each point $r_j$, with $j = 1, \ldots, N_p$, we must determine an analytical model $\tilde{B}_z(I, r_j)$ of the true magnetic field $B_z(I, r_j)$ function of the current vector $I$. In a more formal notation, we have the problem of recovering an estimate $g$ of a mapping $f$ from a set $D$ of data randomly generated:

$$D = \{(x^j, y^j) : \; y^j = f(x^j)\}_{j=1}^{N}. \tag{4}$$

This problem has been studied in the contest of regularization theory [11]. In particular, it has been shown that any continuous mapping $f : R^n \to R$ can be approximated on a compact set with the desidered degree of accuracy by a function $g : R^n \to R$ obtained as linear combination of radial basis functions $\psi(\|x - c^i\|)$, i.e.

$$g(x) = \sum_{i=1}^{M} \lambda_i \psi(\|x - c^i\|), \tag{5}$$

where the scalars $\lambda_i$ are the *weights* and the vectors $c^i$ are the *centers*.

Typical choices for the function $\psi(\|x - c\|)$ are

- $\psi(r) = (r^2 + h^2)^{\frac{1}{2}}$ *direct multiquadric function*
- $\psi(r) = (r^2 + h^2)^{-\frac{1}{2}}$ *inverse multiquadric function*
- $\psi(r) = e^{-\frac{r^2}{h^2}}$ *Gaussian function*,

where $h > 0$ is the so-called *scaling parameter*.

In order to define an analytical expression of the magnetic field generated by a vector of currents $I$ in a given point $r_j$, we propose the following model

$$\tilde{B}_z(I, r_j) = G_{r_j}(I; \lambda_1, \dots, \lambda_M, c^1, \dots, c^M) + K_{r_j}(I; k_0, k_1, \dots, k_4), \tag{6}$$

where

$$G_{r_j}(I; \lambda_1, \dots, \lambda_M, c^1, \dots, c^M) = \sum_{i=1}^{M} \lambda_i \psi(\|I - c^i\|) \tag{7}$$

is a RBF-based contribute and the second term

$$K_{r_j}(I; k_0, k_1, \dots, k_4) = k_0 + k_1 I_1 + k_2 I_2 + k_3 I_3 + k_4 I_4 \tag{8}$$

is introduced to take into account the fact that the magnetic field can be considered, in a first approximation, a linear function of the current.

To define the model (6), once fixed the number $M$ of radial basis functions and the scaling parameter $h$, we had to specify the value of the parameters $\lambda_1, \dots, \lambda_M \in R$, $c^1, \dots, c^M \in R^4$, $k_0, k_1, k_2, k_3, k_4 \in R$. For this purpose, we generated a data set

$$D = \{(I^p, y^p) : \; y^p = B_z(I^p, r_j)\}_{p=1}^{N_p} \tag{9}$$

(the magnetic field value $B_z(I^p, r_j)$ can be obtained by direct measurements, see next section) and solved the following nonlinear least squares problem

$$\min_{\substack{\lambda_1, \dots, \lambda_M \in R \\ c^1, \dots, c^M \in R^n \\ k_0, k_1, k_2, k_3, k_4 \in R}} \sum_{j=1}^{N} [y^p - G_{r_j}(I^p; \lambda_1 \dots, \lambda_M, c^1, \dots, c^M) - K_{r_j}(I^p; k_0, k_1, \dots, k_4)]^2 \tag{10}$$

In particular, for each point $r_j$ we generated, as described in the next section, a data set of $N = 2,000$ elements. In the expression (7) defining the RBF-based contribute we used, as radial basis function, the direct multiquadric function with scaling parameter $h = 0.1$ and we fixed the number $M$ od radial basis functions equal to 4. In this way, we had in problem (10) a total number of 25 free parameters, adequate for the number of data available. Finally, to solve the structured optimization problem (10), we adopted a minimization method, belonging to a class of decomposition algorithms recently proposed [10], suitable for this kind of problems.

## 6. Experimental set up

The currents were produced by four remotely controlled power supplies. Their output was controlled by a PC through four 16 bit digital to analog converters (DAC) in order to generate a magnetic field of 0.1 T. The set of data required for (9) was obtained by the experimental set up of Figure 2.

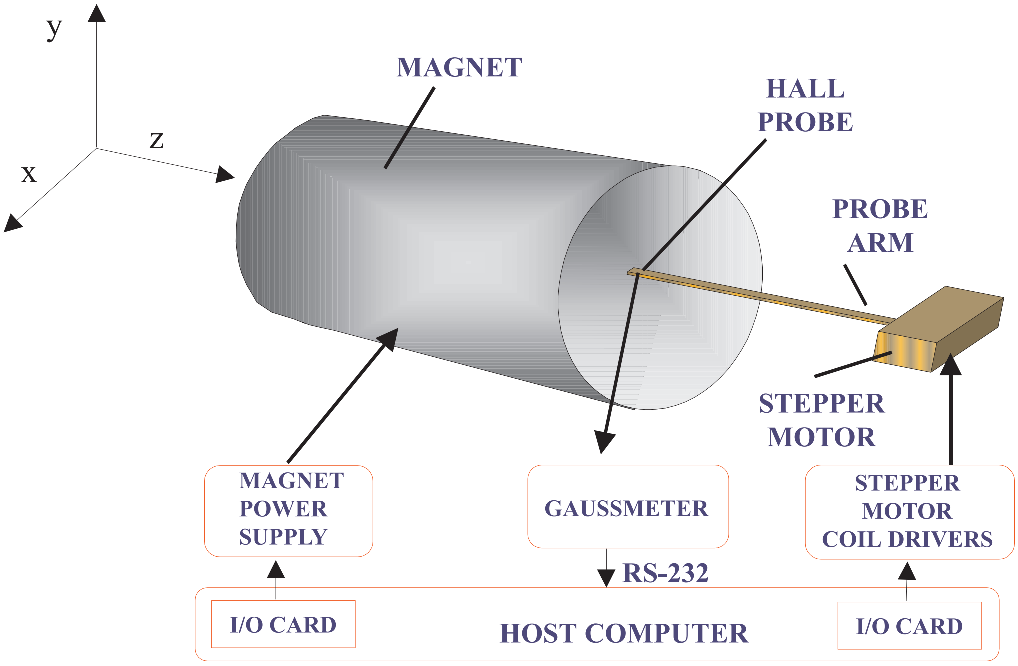


Figure 2: Experimental set up used to alter the current settings and the motor to change the position of the hall probe.

For reasons of symmetry, we considered only points uniformly distributed in the first quadrant of the plane $y = 0$. In particular, we positioned an Hall probe at each of the 9 points ($N_p = 9$) taken for $x = 0,\ 3,\ 6$ and $z = 0,\ 3, 6$cm at $y = 0$, for which we have supposed that assuming verified condition (1) also (2) must hold, and selected 2,000 values of the current $I$. The origin of each axis is placed at the centre of the magnet.

For each position $r_j$, we built the data set (9) by selecting 2,000 values of the current vector $I$ and by measuring the corresponding magnetic field $B_z(I, r_j)$. In order to have a better coverage of the operating region of the magnet, the selected range of the currents was reduced to about 10% of the current values determined by the simulation programs.

Once the data set $D_j$ was obtained, in order to select the parameters that define the model $\tilde{B}_z(I, r_j)$ of the magnetic field given by (6)-(8), we solved problem (10).

Finally, we applied the routine E04DGF of the NAG library for solving the unconstrained minimization problem (3). The components of the computed solution $(I_1^*, I_2^*, I_3^*, I_4^*)$ were inserted as current values through the four DAC and the field distribution given in Figure 3 was obtained. It is compared with the field distribution obtained simply by imposing the current values calculated by the numerical simulation program (Figure 4).

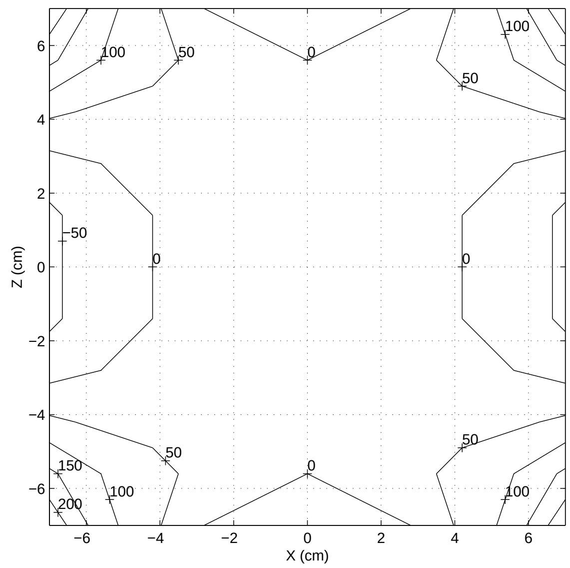


Figure 3: Error map of the magnet field (0.1T), produced by the magnet, expressed in parts per million, using as current values those calculated by the optimization procedure. The directions correspond to those reported in Figure 2, with the origin placed at the centre of the magnet.

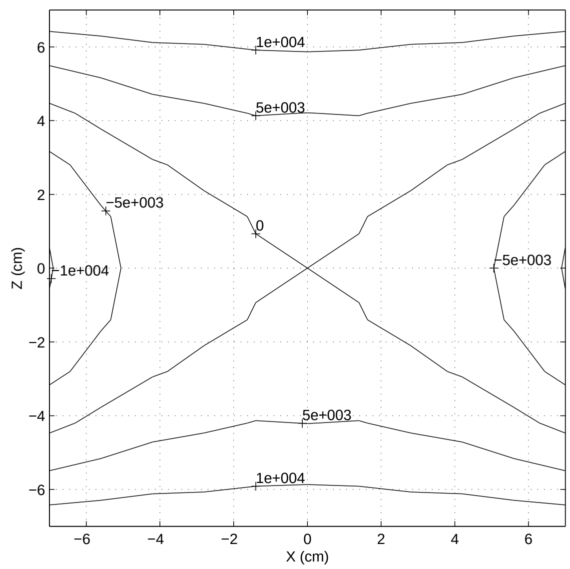


Figure 4: Error map of the magnet field (0.1T), produced by the magnet, expressed in parts per milion, using as current values those calculated using the numerical simulation program. The directions correspond to those reported in Figure 2, with the origin placed at the centre of the magnet.

The field uniformity improvement obtained by means of the optimization technique is impressive. In fact, inside the zone of interest (12 cm at the centre of the magnet) the field inhomogeneities are lower than the Gaussmeter resolution (lower than $5\mu$T). The used Hall probe (Danfisik, Model Group 10) has a field resolution of about $5\mu$T, and this has limited the accuracy both of the magnetic field model and of the final correction. More improvements could be obtained by using a more accurate Gaussmeter, such as NMR Gaussmeter, and by repeating the whole optimization process using 2,000 current values around the previously determined values, using the same sampling points. This would improve the mathematical model of the magnetic field.

## 7. Conclusions

In this work we have proposed a dedicated magnet characterized by a high degree of field uniformity. This has been obtained by controlling the intensities of the currents in four different pairs of coils. The same effect could, in principle, be obtained using a single coil and profiling the coil boundaries. This approach would have the advantage of requiring only one power supply. However, the presence of an iron enclosure and the unavoidable positioning errors of the coil would have required additional shimming coils. The possibility of controlling the coil currents in the present case allowed the use of a shimming procedure, based on an optimization approach, without the necessity of designing and controlling shimming coils of different orders.

The adoption of the proposed optimization-based strategy has determined the achievement of a good uniformity of the magnetic field in about one fourth of the magnet length and about half of its bore. These results seem to be satisfactory, therefore, the magnet could be used in MRI applications.

The choice of a low operating frequency has allowed the design of a magnet which is very convenient in size and weight. Moreover, the use of four pairs of coils and the optimization strategy make possible the use, as a parameter to be controlled, the power required to achieve a given field intensity. This is important ia a compact construction in which power dissipation can become a serious design problem. In a future work, this parameter will also be included in the optimization process.

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